THE MARROW OF THE MATHEMATICKS,

Contracted into a ſmall Compaſs, and made plain and eaſie to the underſtanding of any ordinary Capacity.

CONTAINING

The Doctrines of *Arithmetick*, *Geometry*, *Aſtronomy*, *Gauging*, the Uſe of the *Sector*, *Surveying*, *Dialling*, and the Art of *Navigation*, *&c.* Illuſtrated with ſeveral Cuts for the better Explanation of the whole Matter.

After a New, Compendious, Eaſie Method.

By *W. Pickering*, Merchant-Adventurer.

Licenſed *Rob. Midgley.*

Ex Angulis Latera, vel ex Lateribus Angulos & mixtim, in Triangulis tam planis quam Sphæricis aſſequi, Summa eſt gloria Mathematici. *Sic enim & Cœlum & Terras, & Maria fœlici & admirando Calculo menſurat.* Fran. Vieta.

The Second Edition.

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Collegium Sancti Johannis
Evangelistæ in Academia
Cantabrigiensi

TO THE INGENIOUS READER.

THAT which deters many from attempting, and the most from proceeding in these Studies, is, That they are wrapt up in such a Labyrinth of hard Words, and difficult Terms of Art, that a man may be said first to learn a Language before he can come to the things themselves: And when he hath cracked the shell, and comes to the Kernel, the substance of the matter doth lye in such a Chaos of Confusion, I mean in such a scattered immethodical, and unintelligible way of handling the several parts apart, that how these do lye in the whole Body, or how one piece depends upon another, for a young beginner is extream difficult to understand, if not past finding out: Indeed

ſeveral men have writ excellently well of ſeveral things, and ſome have writ of all the ſeveral parts of the Mathematicks, *Topically, or independently under ſeveral diſtinct Heads; but I have not yet met with any perfect Analyſis of the whole, or ſuch a Co-hærent continued Method in theſe Mathematical Arts as could be wiſhed; In this ſmall piece (therefore) I have indeavoured to remedy theſe inconveniences, and ſupply what is defective. Firſt, For the Terms of Art, and thoſe affected miſterious waies of expreſſion, I have either avoided them totally, or explained them ſufficiently, though whilſt I indeavour to ſpeak more plainly, perhaps by ſome I may be thought to ſpeak leſs Learnedly; but if hereby I attain the end I aim at, I matter not; which is to be underſtood by any that underſtand Common Senſe, and Plain Engliſh.*

In the ſecond place, I have endeavoured to bring the Mathematicks into one entire Method, whereby you may ſee how neceſſarily one part is Co-hærent and dependent upon another; and ſuch as deſire to ſtudy them, proceeding Methodically from

one

one thing to another, they may be greater Proficients in a few dayes, than otherwise in many Moneths, nay perhaps then, in all their Lives, without such a Method. Lastly, In all this, I have used so much brevity, and withal, such perspicuity, that (according to my own weak Judgment and Understanding) there is nothing inserted that could be wanting, nor any thing wanting that need be inserted : In a word, in the reading and studying many Authors, what I found excellent in any of them, I collected, and in most abbreviated, indeavouring in fewer words to make it more plain and easie to understand : And afterwards I have reduced this heap of confused Collections or Miscellaneas into one perfect and entire Method, in every thing indeavouring to make the Body of the Discourse correspondent to the Title-page: Hoping that this will answer both my own and the Ingenious Readers just expectation, and shew both a nearer and clearer way to the understanding the Mathematicks : *That so all may be allured by the plainness and easiness of these Arts, and Persons of ingenuity inamoured on them*

for their uſefulneſs and delight; and having ſo readily paſſed the Porch, and got over the Threſhold, they may have both time and opportunity to build rare Superſtructures upon theſe plain Foundations, and may improve theſe Arts even to miraculous Effects for the finding out many rare Inventions yet unknown to the World; whereas many (otherwiſe excellent wits) thorough their abſtruſeneſs have ſpent their whole time before they could attain to the underſtanding theſe very Rudiments. And hereupon I thought this not an unneceſſary, nor an unuſeful piece of work; and I doubt not but it will be acceptable to ſome, tho' many I know will be ready to cenſure both the Book and the Author; As for my ſelf, I have been ſo accuſtomed to be miſconſtrued in all my Actions, that the wonder would be, not to be ſo in this; and indeed I do ſo little value it, that I can ſmile in diſguiſe, to ſee the moſt ſo much miſtaken, indeavouring in the mean time (as near as I can) to approve my ſelf to Conſcience and Omniſcience. For the Book no doubt, but Carping Momus *may pick ſome Criticiſmes out of it to ſnarl at; but I hope of no great*

moment;

moment; indeed the best Actions of the best men are subject to Detraction, especially their Writings. A detractione neminem esse immunem nisi qui nihil scripserit, *says a Reverend Father of our Church*; Tho. Episc. Covent. *And it is well observed by another*, Facillimum quidem est, & fuit semper, alienum opus reprehendere; sed non perinde facile, aliquid simile aut præstantius conficere: *And all that I shall add, is that of the Poet*,

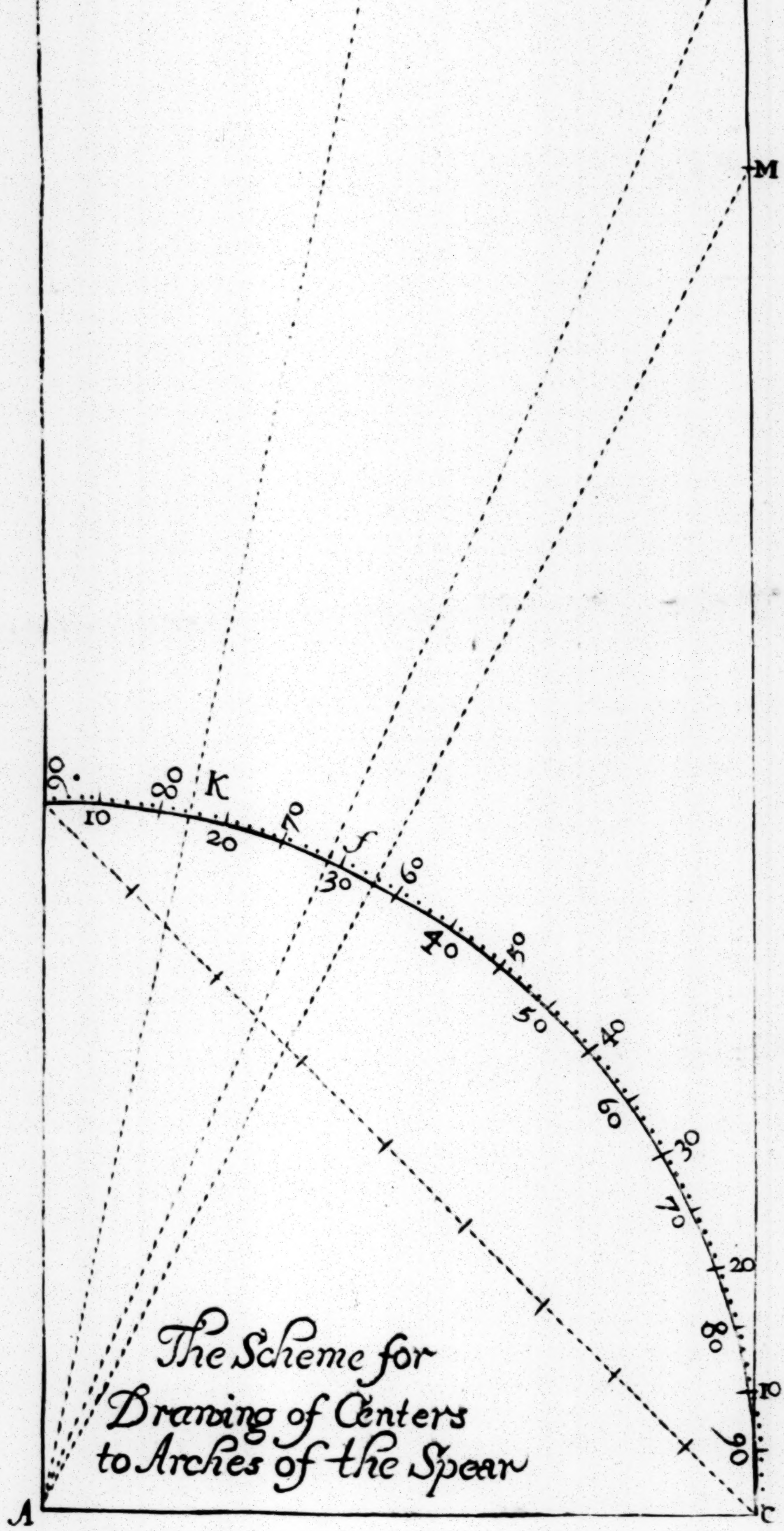
Carpere vel noli nostra, vel ede tua.

Or as it may be Englished,

Either Commend me, or come and mend me.

——-——Si quid novisti rectius istis,
Candidus imperti, si non, His utere mecum.

Vale, Vive bene beateq;



The Scheme for
Drawing of Centers
to Arches of the Spear

THE PRÆFACE OR INTRODUCTION.

THE *LIBERAL ARTS* or *SCIENCES* are reckoned Seven, *Grammar*, *Rhetorick* and *Logick*, *Musick*, *Arithmetick*, *Geometry*, and *Astronomy*. These may not unfitly be divided into Two parts:

First, The *Scholastical*, or Speaking Part, *Grammar*, *Rhetorick*, and *Logick*.

Secondly, The *Mathematical*, or Measuring Part, which comprehends *Arithmetick*, *Geometry*, and *Astronomy*.

Musick which is usually placed in the middle, partakes of a middle or mixt Nature betwixt these Two Generals. For *quoad sonum* it may belong to the former, to wit, the *Speaking Part*, being an inarticulate kind of *Speaking*: But *quoad proportionem* to the

the latter, to wit, the *Measuring Part*, consisting chiefly in due Measures and Quantities.

It is not my purpose at present to speak any thing of those which I call the *Speaking Sciences*, only to shew how proper this Division is by their Definitions you may take them thus; *Grammatica est Ars rectè Loquendi*, *Rhetorica benè Loquendi*, *Logica benè Differendi.*

The Poet distinguishes and commends them thus;

G. *Grata quidem Ratio est concordi voce relata.*
R. *Gratior est Ratio, veniens ratione venusta.*
L. *Grata ter est Ratio veniens ratione probata.*

Of Grammers there is as great Variety as of Languages. *Latine* is a general Language in these Western parts of the World, and the ground of most other Languages in these parts: so is *Greek* in the Eastern: *Hebrew* the Language we shall speak in Heaven say some, and the Original Language of all others say most; And *High Dutch*, as Ancient as the Building of *Babel*, say many.

But

But of all other Grammars and Languages most sutable to this present Subject, is that most incomparably ingenious Invention of Bishop *Wilkins* (that Famous *Mathematician*) which he calls his *Philosophical Language*, with a way of writing the same, called his *Universal Character*, which if it would be as Universally, and as industriously practised, as it is most excellently, and ingeniously contrived: By this means all persons (at least all of any competent capacity) of what Nation or Language soever might converse and correspond with one another as familiarly, as *Adam* and *Eve* in Paradise, or those Eight in the Ark, when the whole World spoke but one and the self-same Language. But enough of These as to my Purpose (to wit) of *Grammar*, *Rhetorick*, and *Logick*.

And for *Musick*, which I call the Meane in this Division, I mean not to run any large Divisions upon it; that Art being rather learn'd by Practice than by Precept; and my Aim being rather to propound Demonstration to the Eye, than Musick to the Ear. Indeed the Musick of the Sphears might have been no improper Theam, but that being grounded upon the old erronious Opinion of their *solid Orbs*, are both disclaimed long since by later Writers. Indeed, I read, that *Philo* the *Jew* saies, that hereby *Moses* was inabled to tarry in the

Mount 40 days, and 40 Nights, without eating or drinking any thing, becauſe he heard the melody of the Heavens: and *Plutarch* ſpeaks as if a man might hear this *Muſick* if he were an Inhabitant in the Moon: But theſe conceits being by the Learned long ſince exploded, I ſhall no further touch upon this ſtring, *viz.* *Muſick*, with the other *Speaking Arts* before-mentioned: But paſſing by all theſe, That which I call the *Mathematical*, or *Meaſuring Part* of the *Liberal Sciences* is the ſubject I intend to treat upon; and (as I apprehend it) may fitly be comprehended in theſe Three, *Arithmetick*, *Geometry*, and *Aſtronomy*: *Aſtronomy* may be ſaid to meaſure the Heavens, *Geometry* the Earth, and both by means and help of *Arithmetick*. This laſt conſiſting chiefly in *The Rule of Three*, the two former in the Reſolution of a Triangle. And when I ſay *Geometry* meaſures the Earth, it is to be underſtood the whole *Globe* of the Earth, containing *Sea* and *Land*, with all other Bodies therein contained, whether *Superficies* or *Solids*: What concerns the Land, may in one word be called *Surveying*: What concerns the *Sea*, is properly called *Navigation*: And not only both theſe, but alſo *Aſtronomy* with theſe may be called the Taking of Heights and Diſtances: Surveying being Taking Heights and Diſtances at Land, Navigation at Sea,

Aſtro-

Aſtronomy in the Heavens, and of Aſtronomy, the moſt Mathematical and Practical part is Dialling, or finding the hour of the day by the Sun, and the hour of the night by the Moon and Stars.

Therefore to ſumm up all in ſhort, the Mathematical part of the Liberal Sciences are three, *viz. Arithmetick, Geometry*, and *Aſtronomy*. The Practical part of theſe are principally three, *Surveying, Navigation, Dialling*. Theſe three are performed by Trigonometrie, or the Reſolution of a Triangle, which derives its name from Three, and conſiſts of Three Angles, and Three Sides, and theſe found by the *Rule of Three*, makes good that old Motto, *Tria ſunt Omnia*.

You ſee then, That the grand Deſign of the Mathematicks is to take Heights and Diſtances at Sea, at Land, and in the Heavens, and to meaſure not only the Heavens and the Earth, but all other Bodies therein contained, whether Superficies or Solids. At Land, to find Heights by Diſtances, and Diſtances by Heights, where either is acceſſible; and both theſe by Angles of Poſition and Interſection of Lines, where both or either of them are inacceſſible. At Sea for Diſtances, to find the Diſtance of a Ship North and South, which is called Latitude, or its Diſtance Eaſt and Weſt, which is called Longitude, or the Diſtance

from any of these Cardinal points, which is called the Rhumb; Lastly, the Distance Run upon the said Rhumb, or upon such a point of the Compass: I am not ignorant in strict propriety of Terms, a Rhumb differs from a point of the Compass. A Rhumb containing two points, *viz.* the point and its opposite point: the former being the whole Diameter, the latter only the Semidiameter of the Compass from the Center, so that there are (to speak according to that nicetie) only 16 Rhumbs, and 32 points of the Compass; but for more plainness and ease to the Learner, I make them here signifie the same thing, being the same in substance. And by an Account kept of these to know the Distance of one Ship from another, or of the same Ship from the place from whence it set Sail, or from the place to which it is bound; and consequently how to direct her course accordingly. In the Heavens, to find the Distances of the Sun, Moon, and Stars from one another, of one Star from another; particularly concerning the Sun, his Distance from the Equinoctial, which is called his Declination: His Distance from any of the 12 Signs which is called his place in the Zodiack: His Distance of Rising and Setting from the due point of East or West, which is called his Amplitude: His Distance at any time from any point of the Horizon

zon, which is called his Azimuth; together with the Distances of all those Cœlestial Bodies, to wit the Planets, the Sun, Moon, and fixed Stars from this Terrestrial Globe: All these may be called Distances in Astronomy: And for Heights, both in Astronomy and Navigation, they are the same; as taking the Height or Altitude of the Sun and Stars at any time above the Horizon which is called their Almicantaras, particularly the Meridian Altitude, either of the Sun or Stars: And thereby to find the Height or Elevation of the Pole: So that by all this you see, that the taking of Heights and Distances at Sea, at Land, and in the Heavens is the most principal, and the most practical part of the Mathematicks: And these are all perform'd by Trigonometry, or by the making and measuring, the framing and resolving of a Triangle, either Plain or Sphærical: Supposing these Heights and Distances to be the several parts, that is, either the sides or Angles of a Triangle. And to find by some of these Parts known, other Parts unknown or *per Data, Quæsita.*

This Resolving of a Triangle, or Trigonometry, resolves all Mathematical Questions, and may be performed by Instruments and Protraction Geometrically: But principally by the Rule of Proportion Arithmetically; worthily carrying the name

of the *Golden Rule*, for the admirable Effects it produceth, especially in these Mathematical Operations. And for the fuller and more ample Explanation of these particulars, I shall proceed to Discourse of them in this Method in 3 Books.

I. In the First I shall Treat of Arithmetick, under the several kinds of it, *viz.* *Common*, *Decimal*, *Logarithmetical*, and *Instrumental*. And in the sequel of the Discourse, as occasion shall be, I shall shew how these may be applyed to the Resolution of a Triangle, or any other Mathematical Question.

II. In the Second Book I shall Treat of Geometry, or any other Mathematical Question. First in general, and of the Three Famous Grametrical Figures, the *Circle*, *Square*, and *Triangle*; especially the last of these. And then Secondly, In particular I shall speak of Geometry, as it consists in Surveying and Navigation; and shew that these are no more than the Resolution of a plain Triangle.

III. In the third and last Book I shall Treat of Astronomy, and therein of Projection of the Sphære, Questions of the Sphære and Dialling, and shew that as Surveying and Navigation are performed by the Resolution of a Plain Triangle, so these by the resolution of a Sphærical Triangle.

THE

The First BOOK.

OF ARITHMETICK.

CHAP. I.

Which as before I do divide into 4 Parts, 1. Common, 2. Decimal, 3. Logarithmetical, 4. Instrumental, *or* Lineal. *Of these in their order.*

COMMON ARITHMETICK to those that are expert in it, especially in working by Fractions, is of singular use, and absolutely necessary; and for the most part of sufficiency enough for Mathematical Operations; and withal it is to be supposed, That no man that attempts the Mathematicks, but in some measure understands those Common Things of *Numeration*, *Addition*,

Subſtraction, *Multiplication*, and *Diviſion*, therefore I ſhall be very brief in them; and only tell you

SECT. I.

Numeration is the Reading of Figures according to the value they receive from the place they ſtand in: The Firſt place ſignifying Units: the Second, Tens: the Third, Hundreds: And the Fourth, Thouſands, according to the enſuing Table.

Numeration

Numeration TABLE.

Units.	1
Tenns.	12
Hundreds.	123
Thouſands.	1234
X. Thouſands.	12345
C. Thouſands.	123456
Millions.	1234567
X. Millions.	12345678
C. Millions.	123456789
Thouſand Millions.	1234567891
X. Thouſ. Millions.	12345678912
C. Thouſ. Millions.	123456789123
Dillions.	1234567891234
X. Dillions.	12345678912345
C. Dillions.	123456789123456
Thouſand Dillions.	1234567891234567
X. Thouſ. Dillions.	12345678912345678
C. Thouſ. Dillions.	123456789123456789
Trollions.	1234567891234567891
X. Trollions.	12345678912345678912
C. Trollions.	123456789123456789123
Thouſand Trollions	1234567891234567891234
X Thouſ. Trollions.	12345678912345678912345
C. Thouſ. Trollions	123456789123456789123456

One Hundred Twenty Three Thouſand Four Hundred Fifty Six Trollions Seven Hundred Eighty Nine Thouſand One Hundred Twenty Three Dillions Four Hundred Fifty Six Thouſand, Seven Hundred Eighty Nine Millions One Hundred Twenty Three Thouſand Four Hundred Fifty Six.

SECT.

SECT. 2.

II. *Addition* is the adding of two or more Sums together, to find the Total.

In like Denominations. In divers Denom.

			D.	M.	Sec.
6789432	Degrees	D.	136	11	9
1903467	Minutes	I	320	10	45
8542079	Seconds	II	429	54	26
17234978	Summa Total.		886	16	20

SECT. 3.

III. *Subtraction* is the Substracting or Deducting of one Sum from another, to find the Remainder.

	D.	I.	II.
As from	2345	36	49
Take	873	18	13
Remainder is	1472	18	36

SECT.

SECT. 4.

IV. *Multiplication* is a Compendious *Addition.*

The TABLE.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	5	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	4	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	54	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Find the Multiplicand in the Top of the TABLE, and the Multiplyar in the side thereof, and in the Square where These meet, is the Product of the Nine Digits; as 7 times 9 is 63, and so of the rest.

In Multiplication the Sum to be Multiplyed is called the Multiplicand, the Sum by which the other is Multiplyed, is called the Multiplyer, and the Number produced by these is called the Product.

Multipli-

Multiplicand	463298765
Multiplyer	23
	1389896295
	926597530
Product	10655871593

To Multiply compendiously by any of the Nine Digits, without charging the Memory.

To Multiply any Number by 2,

Either double it, or ſet it down twice, and add the two ſums together.

To Multiply by 3.

To the Number given, add the double thereof, the ſum of them ſhall be the Product.

To Multiply by 4.

Double the Duplication of the Sum given.

To Multiply by 5.

To the Number given add a Cypher, and the half of that Number is the Product.

To Multiply by 6,

To the Number given add a Cypher, then take the half of that Number, and to it

it add the Number given, the Sum of them ſhall be the Product.

To Multiply by 7,

To the Number given add a Cypher, and take the half thereof, to which half add the double of the Number given, the Sum ſhall be the Product.

To Multiply by 8,

To the given Number add a Cypher, and from thence ſubſtract the double of the Number given, the Remainder will be the Product.

To Multiply by 9,

Add a Cypher to the given Number, and from thence ſubſtract the given Number, the Remainder ſhall be the Product.

To Multiply by 10 add a Cypher, to multiply by 100 add two Cyphers, by 1000 add three Cyphers, and it is done.

If Cyphers be in the middle of the Multiplyer among other Figures, remove the next Figure a place farther where any Cypher comes, and add the Lines together in the ſame order they ſtand.

SECT. 5.

Division is a compendious Subſtraction, wherein the Sum to be divided is called the *Dividend*; the Sum by which we divide the ſame, is called the *Diviſor*; what is produced thereby, is called the *Quotient* and the *Fraction*. *Example.*

```
             21(1
            1091(8
Dividend  467913 8 | 203440 18/23  parts.
Diviſor   2333333    Quot. Fraction.
           22222
```

In dividing any great Sum, to know certainly what Figure to put in the Quotient, and to Divide the Sum by Subſtraction inſtead of Multiplication.

Firſt, Set down your Diviſor, and againſt it the Digit 1, In the next place double it, and againſt that ſet the Number 2. Add the former two Sums together, and againſt that ſet the Figure 3, to that Number add the Diviſor, and againſt that ſet the Figure 4, *&c.* ſo you will have a Table of the 9 Digits, and may ſee againſt every Figure how much the Multiplication of your Diviſor by any Digit will amount unto; ſo that when you ſee your Remainder in

in your Division, look in the Table for the nearest Number less than your Remainder, and that Digit which stands against that Number, you must alwaies put in your Quotient.

To Divide by any Number that hath Cyphers to the Right Hand.

To Divide by 10, cut off the last Figure towards the Right Hand, and the rest of the Figures are the Quotient. To Divide by 100 cut off two of the last Figures, by 1000 three, *&c.* To Divide by 20; cut off the last Figure, and take the half of the Remainder.

As for Example.

To know how many Pounds there is in 243|6 Shillings is the same as to Divide by 20. Therefore cut off the last Figure 6, and take half the rest, that is, half 243, is 121 Pounds, and there remains 1, and the 6 cut off for the Fraction, which is $\frac{16}{20}$, or 16 shillings, and so in any other Sum. The same Reason holds if you be to divide by 30, cut off the last Figure, and take the third part; and if by 40 the Fourth, 50 the Fifth part, and so forwards. The remaining Fraction being alwaies the parts of its respective Integer.

These

These Four, *viz. Addition, Subtraction, Multiplication*, and *Division* (not to mention *Numeration*, which is only the Reading of Figures) do contain the whole Practice of Arithmetick, being judiciously applyed and made use of according to the Nature of each several Question, and by the Invention of Logarithmes these Four are reduced to these Two only, *viz. Addition* and *Substraction*, as you will hear hereafter.

But besides these Essential parts of *Arithmetick*, there are divers other necessary Rules for directing the way or manner of Operation by these.

SECT. 6.

Therefore in the sixth place is that incomparable *Rule of Three*, worthily carrying the Name of the *Golden Rule*, especially, and above the rest to be taken notice of; which yet in the Operative and Practick part of it, is no more but the right Application of Multiplication and Division, that is, by Three Numbers given, to find a Fourth, according to these Two Directions.

First, If the Third Number be greater than the First, and the Fourth required to be greater than the Second, then Multiply the Second by the Third, and Divide the Pro-

Product by the First, the Quotient gives the Fourth Number.

As if 12 *yds.* of Cloth cost 3 *l.* what cost 435 *yds.*

435 *yds.*
3
———
1305

$\cancel{1}\cancel{2}(9$
$\cancel{1}3\cancel{0}\cancel{5}[108\frac{9}{12}$ or $\frac{3}{4}$ *l.*
$\cancel{1}2\cancel{2}\cancel{2}$
$\cancel{1}\cancel{1}$

Secondly, If the Third Number be greater than the First, and the Fourth required to be less than the Second, then Multiply the Second by the First, and Divide by the Third. As,

If 12 men raise a Fort in 8 days, in how many days shall 24 men do it.

8
12
——
96

$\cancel{1}\cancel{0}$
$9\cancel{6}[4$ *dayes.*
$2\cancel{4}$

I. The Direct Rule is, when more requires more, and less less.

II. The Back Rule is, when more requires less, or less more.

For the placing the Numbers right, observe this Rule in the *Direct Rule of Three.*

Three Numbers being given, the Question is annexed but to One, and this must alwaies

alwaies be the Third Number, and that which agrees with this Third in the ſame Denomination muſt be the Firſt, and that which remains the Second. As if the Queſtion were put, If 10 yards coſt 8 *l.* how many yards may we buy for 12*l.* place them thus; If 8 buy 10, what will 12 buy.

SECT. 7.

The next Rule which I ſhall here mention, is the *Rule of Fellowſhip*, which is, when ſeveral Merchants or other Men put in ſeveral Sums together into Stock, and Trading therewith, they either gain or loſe ſo much in the whole; Now the Queſtion will then be, what every particular Adventurer ſhould have of the Profit, or bear of the Loſs according to each mans due proportion. To do this, ſay, If the whole Stock produce ſo much as is the whole Profit or Loſs, then what gives the Firſt mans, Second and Third mans part of Stock ſeverally making each mans particular Stock the Third Number to theſe other Two Numbers, (*viz.* the whole Stock, and the whole Profit or Loſs) and the Fourth will be each particular mans Profit and Loſs.

As if Three Merchants put into Stock 1684 pounds, The Firſt puts in 750*l*, the Second 620*l*, and the Third 314*l*, They Trade

Trade together till they gain 500*l.* The Queſtion is what every one ought to have for his ſhare.

For the Firſt ſay,

If 1684 *l.* give 500 *l.* what gives 750 *l.*

For the Second ſay,

If 1684 *l.* give 500 *l.* what gives 620 *l.*

For the Third ſay,

If 1684 *l.* give 500 *l.* what gives 314 *l.*

SECT. 8.

To find a mean proportional, Multiply one Extream Number by the other, and the Square Root of the Product is the mean proportional: As 72 multiplyed by 32, the Product is 2304, the Square Root whereof is 48: For proof 48 multiplyed by 48, produces the ſame Number 2304, and this is the mean proportional betwixt 72 and 32.

SECT. 9.

To conclude this particular, The Extracting the Square and Cube Root is the hardeſt Leſſon in *Common Arithmetick*: Thoſe which have a mind to do them this way, I ſhall refer them to the Sea-mans *Kalender*, in the latter end of which Book it is plainly

ſet

ſet down: But this way being very difficult and tedious, I ſhall ſhew you very ſhort and eaſie wayes hereafter to do it by *Logarithmes, Gunters Sector, VVingates Rule of Proportion*, or any ordinary Line of Numbers, whereunto I refer the Reader at this time, and ſhall proceed to the Rule of Falſehood.

SECT. 10.

The *Rule of Falſehood* is ſo called, becauſe it teacheth (by feigned Numbers taken at all adventures) to find out the true Number that is demanded; And that 1. Either by one Falſe Poſition: Or, 2. When it requires two falſe Poſitions.

The Rule of one falſe Poſition is this, In ſtead of 3 Numbers given in the *Rule of Three*, by thoſe to find a fourth; In this Rule we have but one Number that cometh in uſe to work by: Unto the likeneſs whereof we muſt feign or deviſe two other Numbers, which ſhall hold the ſame proportion to one another which the third number given ſhall have to the number ſought As for Example. I delivered at Intereſt a certain Sum of Money at 6 *per Cent.* and at the end of 10 years I received 500 *l.* for Principal and Intereſt, I demand how much was the Principal Sum?

To anſwer this, let us feign a Number at pleaſure, and with the ſame, let us make

our

our Discourse even as if it were the principal Sum we seek for: As Suppose I delivered him at the first 200 *l*, which in 10 years at *6 per Cent. Simple Interest* will amount to 120 *l*, which added to the principal 200 *l*, makes together 320 *l*, but this should be 500 *l*: Therefore say by the *Rule of Three*, If 320 *l*. come of 200 *l*, what doth 500 *l*. come off? Multiply the two last Numbers 200 and 500 *l*. by one another, they amount to 10000 *l*, which divide by the first Number 320, and the Quotient is 312½, which is the Sum sought for, or the Principal Sum delivered at Interest at the first. You will find the same number, If you suppose 160 *l*. to come of 100 *l*. then 500 *l*. will in like manner come of 312½ as before: Or any other supposed Numbers which hold the same proportion to one another, which 500 *l*. holds to the Number sought.

2. The Rule of Two False Positions I shall endeavour to make plain on this manner, suppose an 100 Ducates is to be divided amongst 3 Persons, the first to have a certain number of them, the second must have twice so many as the first, abating 8 Ducates, and the third must have 3 times so many as the first, less by 15 Ducates. Now I demand how many every one should have.

First, You may imagine any number at pleasure which you shall call the first Position,

tion, and with the ſame you ſhall work inſtead of the true Number ; and if you ſee you have miſſed of the true Number that you ſeek, then is the laſt number of the work either too great, or too little ; which number you ſhall note with the ſign of more or leſs, and that you call the firſt Error ; now the ſign of more ſhall be noted with this Figure +, the ſign of leſs with this plain Line −. For Example, Suppoſe the firſt man in the Queſtion had 30 Ducats, then by the order of the Queſtion, the ſecond ſhould have 52, the third 75 ; theſe three added, make 157, and I ſhould have but 100 ; ſo that this firſt Error is too much by 57, and therefore I ſet down the firſt Poſition 30 with his Error 57 thus, $30 + 57$.

The next thing to be done is to ſuppoſe another Number, which you muſt call your Second Poſition, and note its exceſs or want of the true number, and that you call the ſecond Error. As for Example, Suppoſe you take 24 for the firſt Man in the Queſtion, then by the order of the Queſtion, the ſecond was to have 40, and the third 57 ; theſe three added make 121, and I muſt but have 100, ſo the ſecond Error is too much by 21, and therefore I note it down under the former thus, $\left\{ \begin{array}{l} 30 + 57 \\ 24 + 21 \end{array} \right\}$ Then

you

you muſt Multiply the firſt Poſition 30 by the ſecond Error 21 Croſswiſe; and again the ſecond Poſition 24 by the firſt Error 57, (and this muſt alwaies be obſerved) and note down the two Products, *viz.* 630, and 1368; then obſerve this Rule.

Both ſigns alike Subſtraction *do require,*
But unlike ſigns Addition *will deſire.*

And accordingly, If both ſigns be alike, that is, both too much, or both too little, you muſt ſubſtract the leſſer Product from the greater; as in this Example, 630 from 1368, and note the Remainder, which is 738, and this muſt be your Dividend. Again, you muſt ſubſtract the leſſer Error from the greater, *viz.* 21 from 57 Reſts 36, this muſt be your Diviſor. Therefore Divide 738 by 36, the Quotient ſhall be the true Number ſought, or the Number of Ducats which the firſt man was to have, *viz.* 20 $\frac{1}{2}$, and conſequently the ſecond man had 33, and the third 46 $\frac{1}{2}$, making in the whole 100.

But note, that if the ſigns of the two Errors had been unlike, that is, if one of the Errors had been too much, and the other Error too little, then inſtead of ſubſtracting the one Product from the other, you muſt have added the one to the other; and likewiſe you muſt have added both the

Errors together, and by the Sum of thoſe Errors you muſt have divided the Total Sum of both the Products, and the Quotient ſhould then have been the true Number ſought. As for *Example.*

If the two Poſitions were 24 and 20, you will find the firſt Error will be 21 too much, and the ſecond will be 3 too little. Therefore multiply 24 by 3, produceth 72. Likewiſe, multiply 20 by 21 produceth 420, and becauſe the ſigns of the Errors are unlike, add them 492, which ſhall be your Dividend; and again add the leſſer Error 3 with the greater 21, makes 24 for your Diviſor; and laſtly, Divide 492 by 24, the Quotient is $20\frac{1}{2}$ as before, for the firſt mans ſhare; and conſequently the ſecond muſt have 33, and the third $46\frac{1}{2}$, making up together 100, as in the former *Example*: And this I hope is ſufficient for the underſtanding of this excellent *Rule of Falſehood*, or of *Falſe Poſitions*, in the ſeveral kinds of it; though I might have added variety of other *Examples*: But to avoid prolixitie, and for brevities ſake theſe may ſuffice.

CHAP.

CHAP. II.

Of Decimal Arithmetick.

IN the Second place most Mathematicians do use and extol *Decimal Arithmetick*, and do accommodate all their Scales, Rules, and Lines to this kind of *Division*, and not without cause, whereby they have this advantage, that the Fractions which in Common Arithmetick are of divers Denominations, in this are all Decimal Fractions; that is, $\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1000}$ parts of any thing, and by adding Cyphers, they may bring them to smaller Fractions, and this makes the working by Fractions far more easie and exact.

To Reduce ordinary Fractions into Decimal.

Multiply the Numerator (or the Number above the line) by 100, or 1000, and Divide the Product by the Denominator (or that Number which is under the Line) and the Quotient shews the Numerator of the Decimal Fraction; As $\frac{3}{4}$ multiply 3 by 100 (by adding 2 Cyphers) makes 300, Divide 300 by 4, and the Quotient will be 75, so

that $\frac{3}{4}$ in Common Fractions is $\frac{75}{100}$ in Decimal.

CHAP. III.

Of Logarithmetical Arithmetick.

IN the third place the Invention of *Logarithmes* by my Lord *Napier* is an excellent Invention; for what is done by Common or Decimal Arithmetick by Multiplication and Division, are done by these artificial Numbers, by *Addition* and *Substraction* only: The Square Root is extracted by Bipartition, and the Cube Root by Tripartition; to explain these more particularly in all the necessary parts of Arithmetick, as first Numeration in Logarithmes, or how to read them.

SECT. I.

In the Table of Logarithmes you will find against every Natural number (from 1 to 1000)) its artificial Number or Logarithme; so if you seek the Logarithme of 50, is 1.698970, and if you would find the natural number to this Logarithme 1.62249, over against it you find 42, and so the like of

of all others. And what is ſaid of Logarithmes, as to numbers, the like is to be underſtood all along of Natural Sines and Tangents, and their Artificial Logarithmes, as you may perceive by the Tables.

By the Table of Logarithmes, to find the Logarithme of a Number conſiſting of 4 places, as ſuppoſe 5628, the ordinary Numbers reaching only to 1000.

Firſt, Take the three firſt Figures, *viz.* 562, and find that in the firſt Colum under the letter *N*, then for the laſt Figure, which is 8, Find amongſt the great Figures in the head of the Table, and in the common Area or meeting of theſe two lines you will find .750354, before which I place 3 for the proper Characteriſtick makes 3·750354 the Logarithme of 5628, which was required.

Now beſides the Logarithme it ſelf, there is alſo to be placed before it his proper Characteriſtick, *viz.* a Figure conſiſting of an Unite leſs than the number of digits or places in the ſaid number, whoſe Logarithme is to be expreſſed: As the Characteriſtick of any number conſiſting of one Figure is 0, of two Figures 1, of three Figures 2, &c. as the Logarithme of 415 is .618048, and becauſe it conſiſts of three Figures, its Cha-

racteristick is 2, placed thus with a Comma 2,618048.

To find the Logarithme of a proper Fraction, as of $\frac{2}{3}$. First I find the Logarithme of its Denominator 3 to be 0,477121, then I find the Logarithme of its Numerator 2 to be 0,301030: And substracting the latter from the former, there remains the Logarithme of $\frac{2}{3}$, which is 0,176091.

SECT. 2.

To Multiply by Logarithmes is to add one Logarithme to another, and that does the same as to multiply the natural Numbers: As to Multiply

24	by	Log.	1.380211
32	is	Log.	1.505150
48			2.885361
72			
768			

Over against which Logarithme you will find in the Table the natural number 768

SECT. 3.

To Divide, is to Substract one Logarithme from another, and that does the same

ſame as to divide one natural number by another.

As to divide	36	whoſe Log.	1.556302
by	12	whoſe Log.	1.079181
is	3	Reſts	0.477121

Which Logarithme you will find to anſwer to the natural Number 3, the Quotient as before.

SECT. 4.

To Perform the *Rule of Three* by Logarithmes. As in *Common Arithmetick* we multiply the Second Number by the Third, and divide by the firſt (which is the *Direct Rule of Three*) ſo you muſt add the Logarithmes of the ſaid Second Number and Third together, and from the Total ſubſtract the Logarithme of the firſt number, and the remainder is the fourth number ſought. As for *Example.*

If 12 give 4, what gives 60.

60		
—		
(0	Log. of 4 is	0.602060
240	of 60 is	1.778151
122		
x	Added are	2.380211
——	Subst. the third	
20	Numbers Log.	1.079181
	Rests	1.301030

Which in the Table you will find to be the Logarithme of 20, the Quotient by Common Arithmetick as above.

SECT. 5.

To Extract the Square Root (which is the same as to find a mean proportional between 1 and the numbers given) therefore to do this observe, That half the Logarithme of the Number given is the full Logarithme of the Square Root: So the Log. of 144 is 2.158362, half whereof is the Square Root, *viz.* the Log. of 12, 1.07918.

SECT. 6.

To Extract the Cube Root is likewise no more but to take the third part of the Loga-

Logarithme of any Number, whose Cube Root is demanded, which third part is the Logarithme of the same Cube Root: As for *Example*. The Log. of 64 is 1.806180, the third part whereof is 0.602060, which you will find to be the Logarithme of 4, which is the Cube Root of 64; the reason is, because the Cubiq; Root is always the first of two mean proportionals betwixt 1, and the number given:

Note that any Number multiplyed into it self, the Product is the square number, and the Square Root is that same first Number; as 8 multiplyed by 8 gives 64 his Square, and his Root 8. And multiply a Square Number by his Root gives you the Cube; as 64 multiplyed by 8 gives 512 the Cube Number, his Root 8.

SECT. 8.

To find a mean proportional between two Extream numbers given. Add the Logarithmes of the two Extream numbers together; the half of the Sum shall be the Logarithme of the mean proportional required. As to find a mean proportional betwixt 8 and 32, which shall bear the same proportion to 32, as 8 does to it.

The Logar. of 8 is		0.903090
The Logar, of 32 is		1.505150
Added together is		2.408240
Half whereof is		1,204120

The Log. of 16, the mean proportional; for as 16 is to 8, ſo is 32 to 16.

Thus you have Numeration, Multiplication, and Diviſion by Logarithmes: Alſo the Rule of Three, The Extraction of the Square and Cube Root, and to find a mean proportional between two Extream Numbers given.

He that deſires to know more of theſe, or more at large concerning theſe, I refer him to the latter end of *Gunters* works, from whence theſe are for the moſt part collected.

CHAP. IV.

Of Inſtrumental Arithmetick.

IN the Fourth place I proceed to that kind of Arithmetick, which I call Inſtrumental or Lineal, becauſe it is wrought upon Lines and Mathematical Inſtruments. Thoſe which I ſhall mention are theſe Three, Mr.

Gunters

Gunters Sector, Mr. *Wingates* Rule of Proportion, and Mr. *Seth Partridge*'s Sliding Rule, being only the Lines of Numbers, Sines, and Tangents diverſly improved.

SECT. 1.

The Excellency of *Partridge*'s Sliding Rule conſiſts in this, That one may work upon it without Compaſſes, and after the ſame be right ſet, it often reſolves ſeveral queſtions at once, without altering the poſition; only thoſe which I have ſeen, are ſet to ſo ſhort a Radius, which makes it leſs exact, and more ſubject to Error. I ſhould adviſe the Reader to buy the Book, as being very plain, and for the accommodating of it to any other Lines of Numbers, Sines, and Tangents, which may be to a larger Radius, and to uſe Compaſſes, obſerve this general Rule, That what is Termed in the uſe of that Inſtrument, [Set ſuch a number on the Firſt to another number on the Second, Then againſt the Third Number on the Firſt, is the Fourth on the Second.] The ſame upon the plain lines of Numbers, Sines and Tangents, with a pair of Compaſſes, is thus worded [Extend the Compaſſes from ſuch a number to ſuch a number, (or from the firſt number to the ſecond) The ſame Extent the ſame way will reach from the Third Number to a fourth required: This difference

difference obſerved, the ſame Book and Directions will ſerve for either this Rule, or any other line of Numbers, Sines, and Tangents whatſoever.

SECT 2.

Next as to Mr. *VVingates* Rule of Proportion, Its excellency conſiſts in this, that it is done to a very large Radius, and very neat diviſions, and therefore ſo much the more exact.

Likewiſe by this you have the Square and Cube Roots by inſpection only, and the reaſon is, becauſe to a very large Line of Numbers, called by him the great Line of Numbers there is adjoined a line of Numbers twice repeated to find the Square Root and mean proportions, called by him the mean line of Numbers, and likewiſe a line of Numbers 3 times repeated, called his little line of Numbers, and this applyed to the aforeſaid great line of Numbers finds the Cube Roots, and both theſe by inſpection only.

In other principal Reſpects both theſe Rules do agree with the lines of Numbers, Sines, and Tangents upon the outward edge of *Gunters* Sector, he being the firſt Author and Inventor thereof; therefore having hinted to you wherein the particular Excellencies of theſe other two Rules do

consist, I shall shew you the general Use of these Lines of Numbers, Sines and Tangents upon the Sector (being the same in substance with the two former, only differently improved) as I said before.

SECT. 3.

Therefore in the third place *Gunters* Sector I account the most excellent Instrument of all other, and most general for all Uses, Intents, and Purposes whatsoever; not only in working all proportions, but it may also easily be improved to an excellent Quadrant and Theodolite, thereby to take Heights and Distances, and by that means to become a *Panorganon*, an Universal Instrument to do any thing or every thing both for Calculation and Observation: But in this place I intend only to insist upon what it hath Common with *Partridge*'s Sliding Rule, and *VVingate*'s Rule of Proportion: To wit,

The Lines of Numbers, Sines, and Tangents.

Then to shew you,

I. *Numeration* upon the Lines or the reading of them, which is on this manner. The Line of Numbers consists usually of two parts, the former and the latter, each numbred with these Figures, 1.2.3.4.5.6.7.8.9.1. being the same Line twice repeated.

peated. Now theſe Figures according as you have occaſion, may ſignifie either ſingle Units, Tens, Hundreds, Thouſands, Ten Thouſands, or any other imaginable Number, by Mr. *Wingate* called Primes, Tens, Centeſmes, Millains, *&c.* And the ſmaller diviſions ſignifie proportionably in order to what their Primes are taken for. And in reference to the firſt and ſecond part of this Line take this Rule. If the Primes that is the large Diviſions where the Figures are plac't do in the former part ſignifie Units, only the ſubdiviſions are $\frac{1}{10}$ Tenths parts of the ſame Units, *&c.* accordingly if ſmaller Diviſions happen under theſe: And the Primes in the ſecond or latter part of the Line ſignifie Tens to the former Units, and Hundreds, if the firſt Primes ſignifie Tens; Thouſands, if the firſt ſignifie Hundreds, *&c.* Now to give one Example for all, The firſt Prime in the ſecond part is divided firſt in 10 parts, and thoſe Tens are again really divided into Ten more, and theſe laſt at leaſt ſuppoſed to be divided into Ten more ſtill; ſo that ſuppoſe a man would number upon the Line 11575. The firſt part of the Line ſignifies the Firſt Figure 10 Thouſand. The firſt Prime in the ſecond part ſignifies the ſecond Figure 1000. Five of the larger Diviſions into Tens ſignifie 500, the ſeven ſubdiviſions

divisions into other Tens, signifie 70, and adding to these half of the next small Division by Estimation, signifies the odd 5, so at that very place you have 11575, and so of any other Number, whether bigger or lesser.

For the Line of sines, 'tis easie being numbred by 10. 20. 30. 40. 50. 60. 70. 80. 90. and the former part with 1. 2. 3. 4. 5. 6. 7. 8. 9. These signifying single Degrees, and the subdivisions first into 6, each signify-10 Minutes, and those again divided into halfs, which signifie 5 Minutes, the rest supplyed by Estimation or Supposition: In the second part they are first divided into Ten larger parts, which signifie each one Degree, and those Ten subdivided into 6, which signifie each 10 Minutes, *&c.*

And what is said of the Line of Sines, the same may be said of the Line of Tangents, signifying also Degrees and Minutes, and proceeding forwards to 45, at the end of the Line, and so backwards to 90, where you begun, as you will see signified by Figures to that purpose.

Having as plainly as I can expressed *Numeration* upon the Rule, in the next place I come to shew you the next considerable Rule in *Arithmetick*, upon these Lines: To wit,

II. *Mul-*

II. *Multiplication*: And to Multiply, Extend the Compasses upon the Line of Numbers from 1 to the Multiplicator. This done, the same Extent the same way will exactly reach from the Multiplicand to the Product required: As for *Example*, To multiply 42 by 33, extend your Compasses from 1 to 33, the same Extent will reach from 42 to 1386; so to multiply 18 by 4, Extend the Compasses from 1 to 4 in the former part of the Line, and the same Extent in the second part will reach from 18 to 72. The like is to be understood of all other Questions in Multiplication: Only take this other *Example*.

In Lands and Houses sold at so many years purchase. To find what the value of the whole will be by the Line of Numbers. The Extent of the Compasses from one to the number of years, will reach the same way from the yearly Rent, to the Sum of the Purchase. As the Extent from 1 to 20 will reach from 10 to 200, which shews, that 10 *l. per Annum* at 20 years purchase, is worth 200 *l.* and so of any other.

III. *Division*: To Divide by the Lines, Extend your Compasses upon the Line of Numbers from the Divisor to One: This done, if you apply the same Extent the same way from the Dividend, the move able point will fall upon the Quotient. As to Divide

Divide 12 by 3, Extend the Compasses from 3 to 1, the same Extent will reach the same way from 12 to 4, which is the Quotient.

IV. To Perform the *Rule of Three*, or Three numbers being given to find a Fourth, Extend the Compasses from the First to the Second, and that Extent the same way will reach from the Third number to the Fourth. As to take the same Example as in Logarithmes: If 12 give 4, what gives 60. Extend the Compasses from 12 to 4, the same Extent the same way will reach from 60 to 20, the number required.

V. To Extract the *Square Root* is the same as to find a mean proportional betwixt 1, and the number given; therefore divide the said distance betwixt 1 and the number given into two equal parts, and at that point you will find the Square Root. As for *Example*, the middle distance betwixt 1 and 9, you will find to be at 3, which is the Square Root of 9, for 3 times 3 is 9.

Now by Mr. Wingate's *Rule to find the Square Root by inspection only, take this Rule.*

When the Figures of the Number given are even, as when the Number consists of 2 4 6 8 Figures, look the same number in the first part of the mean Line of Numbers, and

and juſt over againſt it at the ſame point upon the great Line of Numbers (or indeed upon that Line of Numbers beginning in the middle, for this together with the other makes but up the great Line of Numbers compleat) there you have the Square Root by inſpection only; as 144 I find at the point *S*, and over againſt it 12, which is the Square Root thereof.

VI. To Extract the *Cube Root* is to find 2 mean proportionals betwixt the Number given, and 1: Therefore upon the Line of Numbers divide the diſtance betwixt the number given, and 1 into three equal parts, and the firſt of thoſe 3 parts from 1 is the Cube Root. For *Example*, Extend your Compaſſes from 1 to 4, and that will divide the diſtance betwixt 1 and 64 into three equal parts, therefore 4 is the Cube Root of 64, and to prove it, multiply 4 by 4 makes 16, and 16 multiplyed again by 4 makes 64, the Cube number, its Root 4.

Now to do this upon Mr. Wingate'*s Rule, by Inſpection only, Take this Rule.*

When the Number propounded conſiſts of theſe Figures, *viz.* of 1. 4. or 7. find it in the firſt part of that which he calls the little Line of Numbers (being the Line of Num-

Numbers three times repeated.) If it consist of 2 5 or 8 Figures, then find it in the second part, or if of 3 6 9, then find it in the third part: And this done, observe, That at the very same point, upon the great Line of Numbers, you shall find the Cube Root you look for. As for *Example*, To find the Cube Root of 64, because the number consists of 2 Figures, therefore look for 64 in the second part of the little Line of Numbers, and over against it exactly in the great Line of numbers you find 4, which is its Cube Root.

VII. To find a Mean Proportional betwixt two Extream Numbers, is only to divide the space betwixt the Extream numbers into two equal parts, or by *Wingates* Rule, Extend the Compasses upon the mean Line of numbers, from one of the Extream Numbers to the other. This done, the same Extent applyed upon the great Line of numbers from either of the numbers towards the other, the moveable point will fall in the middle betwixt them; *viz.* upon the point representing the mean proportional required. *Example*, 8 16 32.

Thus you have all the necessary parts of Arithmetick performed by the Lines of Numbers, Sines, and Tangents, and what is said as to Numbers only, the like will hold as to Sines and Tangents alone, or these severals

verals mixt, as between Sines and Numbers, Tangents and Numbers, or Sines and Tangents; as the nature of the queſtion ſhall require. He that deſires to know more concerning theſe, I ſhould adviſe to buy Mr. *Wingates* Book, called his Rule of Proportion, wherein you have the Rule it ſelf in paper, which you may get paſted upon wood; and will be no great coſt, and will ſhew you the whole Myſtery of it; Alſo I ſhould adviſe to buy *Seth Partridges*'s Book, ſhewing the uſe of his Sliding Rule, being plainer than *Wingates*, only obſerving the Rule before given for applying the Queſtions, ſo as to be wrought upon a plain Line of Numbers, Sines, and Tangents, and a pair of Compaſſes.

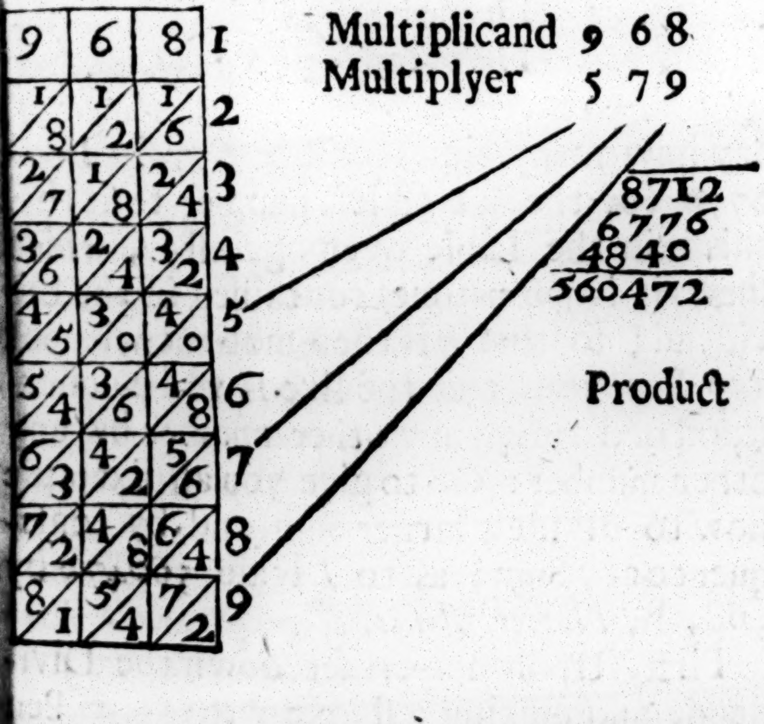
SECT. 4.

To conclude what may be ſaid in reference to *Inſtrumental Arithmetick*, I think it not amiſs to give you a ſhort Account of *Napiers Bones*, whereby Multiplication and Diviſion in *Common Arithmetick* is performed by Addition and Subſtraction only without charging the Memory.

Napiers Bones then are nothing elſe but *Pythagoras* Table with divided Lines. For Reading the Bones begin alwaies on the right hand, and take them as they lye in each Diagonal Line, and if there be two Figures

Figures in the ſame Diagonal, make but one of them by adding them together; and if they exceed 10, take 1 to the Figure of the next Diagonal immediately following. As for *Example.*

Againſt 9 you find in the firſt Diagonal 2 by it ſelf; in the next 7 and 4 which makes 11; therefore ſet down 1, and carry one to the next Diagonal, which is 5 and 1, and 1 you carryed makes 7, and in the laſt Diagonal ſtands 8 by it ſelf, ſo they are to be read 8712, and ſo of any other number whatſoever.



In Multiplication place alwaies the Multiplicand in the Top of the Bones, and against every Unite of the Multiplyer in the side, you will find the particular Product of the same Unite, which added together, as in the *Example*, makes up the whole Product as before.

For *Division* place the Divisor in the top of the Bones, and by finding the nearest less number to the Dividend, you may find how often the Divisor is contained therein.

As for Example.

Divisor 968
Dividend 6776

Say how many times 968 in 6776. Look 6776 (or the nearest less number to it) but this is in the Table over against 7, which shews it is just 7 times contained in the Dividend; so that 7 is the Quotient without any Fraction, and the like is to be understood of dividing any other number by any other number: Or to give you an Example how to divide a larger Sum, and by consequence any Sum; as to Divide 560472 by 968, by *Napiers Bones.*

First, Upon Paper set down the Dividend, and making a stroke with your Pen on the left hand, there place the Divisor,

and

and making another ſtroke with your Pen on the right hand, there place the Figures of the Quotient, then obſerve upon the Dividend how far the Diviſor will reach, as in this Example, to the Figure 4; there make a prick; Then having tabulated the Bones, placing the Diviſor in the Top as before, find out the neareſt leſs number to 5604. which you will find to be 4840. which you will find againſt the Figure 5, therefore put 5 in the Quotient, and place this 4840 under the Figures of the Dividend, and ſubſtract it from 5604, there remains 764, to which add the next Figure in the Dividend, *viz.* 7. (making a prick under the ſaid Figure in the Dividend, to know how far you are gone in the operation) Theſe together make 7647: Then upon the Bones look for the neareſt leſs number to 7647, and you will find it 6776, and over againſt it the Figure 7, therefore put 7 in the Quotient: And Subſtract this 6776 from 7647, the Remainder is 871, to which add the next Figure in the Dividend, *viz.* 2, and theſe make together 8712: Therefore on the Bones look the neareſt leſs number to 8712, but you will find the exact number it ſelf ſtanding againſt the Figure 9, therefore put 9 in the Quotient, and nothing remains, elſe what had remained, had been the Fraction, and this will

will make plain any other operation in Division upon *Napiers Bones*, and therefore I ſhall need to add no more.

968 (560472 (579
4840
———
7647
6776
———
8712

And thus much ſhall ſuffice to have ſpoken for the underſtanding of *Napiers Bones*. As alſo for what I ſhall ſay to the Firſt Part of the *Mathematicks*, conſiſting in *Arithmetick*, and the four ſeveral kinds of it, as it is diſtinguiſhed into *Common*, *Decimal*, *Logarithmetical*, and *Inſtrumental*; and here I intended to have ſhown the Uſe of the Reſt of the *Sector* (beſides the Lines of Numbers, Sines and Tangents): But upon more mature conſideration I find it more properly to be *Geometrical*, then *Arithmetical*, and therefore by way of Tranſition it will lead me by the hand (as it were) to the Second Part of the *Mathematicks*, (or the Second *Mathematical Liberal Science*) to wit, *Geometry*, where you may expect an Explanation of the reſt of the *Sector*, as alſo the way of *Protraction* by Scale and Compaſs.

O F

The Second BOOK.

OF GEOMETRY.

PART I.

GEO METRY is the Second *Liberal Science* which may be properly called *Mathematical*, and may be considered under a double Notion. First in General, as it makes and measures Geometrical Figures. Secondly according to the proper and particular Etymology of the Word, which is derived from γῆ *Terra*, & μέτρον *Mensura*, The Measuring of the Earth (that is as I have elsewhere explained it) the whole Globe of the Earth, containing Sea and Land, that which concerns the Land being called *Surveying*, and that which concerns the Sea, *Navigation*.

I. *But first of* Geometry *in General.*

All Bodies whatſoever which come within the Compaſs of *Geometry*, or are to be meaſured, may be diſtinguiſhed into theſe Two kinds. Firſt, *Superficies*. Secondly, *Solids*: And theſe are repreſented by *Geometrical* Figures, which Figures being protracted or laid down in due Proportion and Quantities anſwerable to the ſeveral Bodies they repreſent. The Bodies themſelves are Meaſured by the ſame Rules as theſe Figures are which repreſent the ſame.

Now theſe Figures are as various, as there are Superficial or Solid Bodies of various and different ſhapes and magnitudes; inſomuch as in this reſpect they may be ſaid to be infinite, ſome Regular, but the moſt Irregular. All which, notwithſtanding of what ſhape or Irregularity ſoever, may be reduced to ſome of theſe three Regular *Geometrical* Figures, the *Circle*, *Square*, and *Triangle*. Therefore in this place I ſhall ſhew you

1. How theſe three Figures are produced, protracted, or made.

2. How they may be reduced to one another, or any other to theſe or ſome of theſe. And,

3. How these, and consequently all other Geometrical Bodies or Figures are to be measured, whereby you will see, that as *Arithmetick* is principally comprized in the *Rule of Three*, so *Geometry* and *Astronomy* may be comprehended in the Resolution, or the making and measuring of a Triangle, called *Trigonometry*.

First then, How these three Figures, the Circle, the Square, and the Triangle are produced, and so they will be found to derive their Original from these three *Geometrical* Principles.

1. A point which is void of all magnitude, and hath neither length nor Breadth.

2. A Line which is a Length without Breadth, drawn from one point to another, and may be either strait or crooked.

3. An Angle which is made by two Lines meeting in a point, that point being called the Angle, or the Angular Point: So that as the *Terms* or *Limits* of a Line are Points, and the *Terms* or *Limits* of Angles are Lines, so the *Terms* or *Limits* of Superficies are both Lines and Angles, and the *Terms* or Limits of Solids are Superficies.

Again, a Point hath neither Length nor Breadth; a Line hath Length, but no Breadth; a Superficies hath both Length and Breadth; a Solid hath both Length, Breadth, and Thickness.

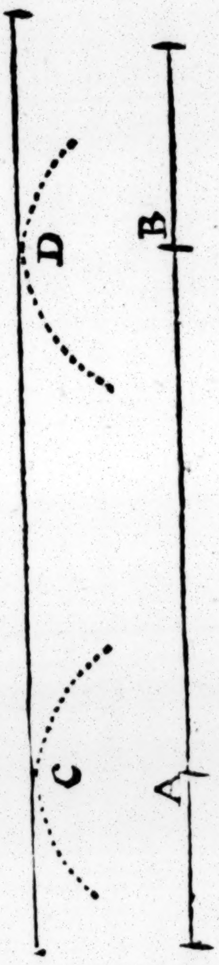
CHAP. I.

BUT before I ſpeak particularly of theſe three *Geometrical Figures*, I ſhould neceſſarily lay down ſome few *Geometrical Problems* becauſe (as a Circle is made of a Line drawn equally diſtant on all ſides from a Point called its Center.) So the Square and the Triangle are made up of Lines and Angles, and therefore there will be ſeveral times mention made of Parallel Lines, and Perpendiculars: And alſo in the two latter Figures there will be occaſion for laying down of Angles, *&c.* I ſhall therefore in this place (once for all) lay down theſe few *Geometrical Problems*, not to trouble you with any more but thoſe abſolutely neceſſary.

SECT. I.

First, *To draw a Line Parallel to any other Line given at any diſtance required.*

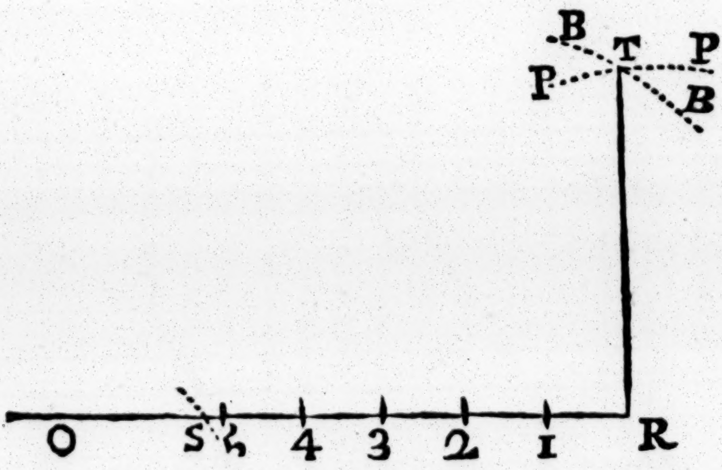
Let the Line given be AB, unto which it is required to draw another Line Parallel thereunto at the diſtance AC, or BD. Open your Compaſſes to the diſtance AC or BD, and place one Foot in A with the other deſcribe the Arch C, alſo place one Foot in B, and with the other deſcribe the Arch D, then draw the Line CD, ſo that it may only touch the Arches C and D, ſo ſhall the Line CD be parallel to AB, as required.



SECT. 2.

To Erect a Perpendicular on the end of a Right Line given.

First, Divide the Line OR from R in any 5 Equal parts, as in the Example 1. 2. 3. 4 5. Then take with your Compasses the distance from R to 4. and placing one Foot in R, with the other describe the pricked Arch PP, then take with your Compasses the distance from R to 5, and placing one Foot in 3, with the other describe the pricked Arch BB, and from the point where these two Arches intersect one another to the point R, draw a Right



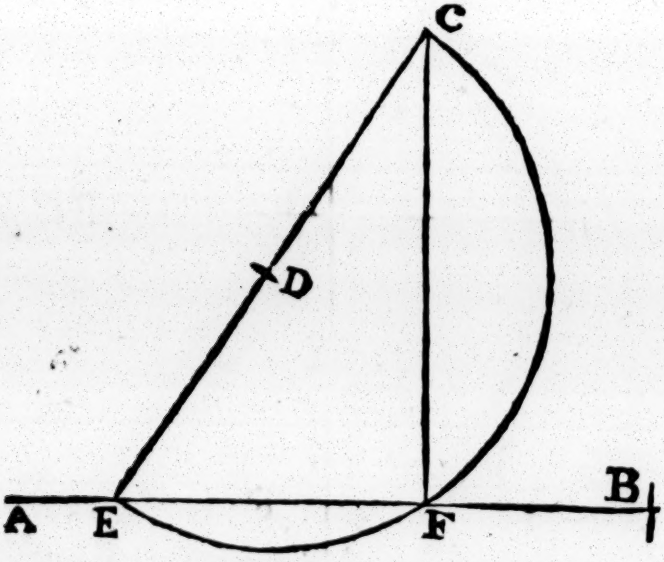
Line, and that will be a true Perpendicular; or open your Compasses to any convenient Distance, and set one Foot in the point

point R, with the other make a mark at pleaſure, as ʘ, then keeping the Compaſs point in ʘ, with the other deſcribe an Arch as BB, as alſo another Arch to cut the given Line in *S*. Laſtly lay a Ruler from *S* to ʘ, will croſs the Arch BB in T, and a line drawn from T to R will be the perpendicular required.

SECT. 3.

How to let fall a Perpendicular from any point aſſign'd upon a Right Line.

Let the Point given be C, and to let fall a Perpendicular upon the Line AB. Firſt from C draw a Line by chance, as CE, which divide into two equal parts at *D*,



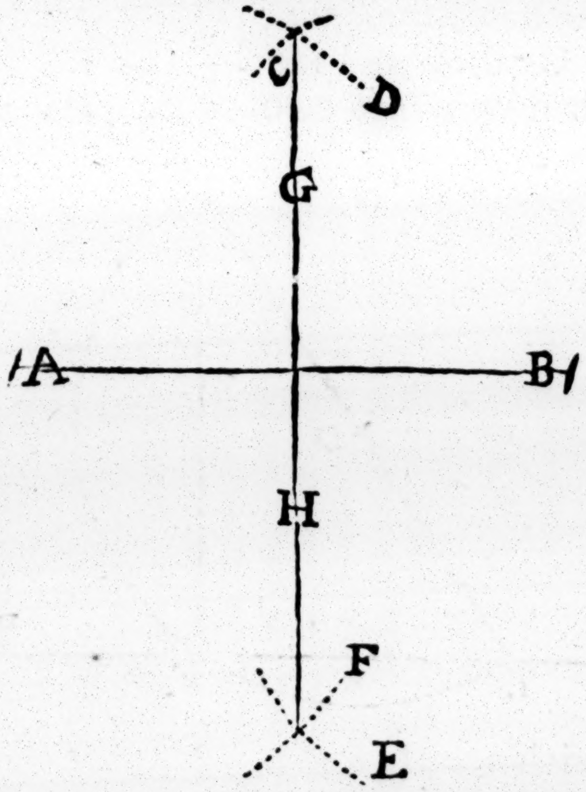
and with the Diſtance D C deſcribe the Semicircle CFE, and note where the ſame

cuts the Line AB, and from that point A, to the point C, draw the right Line C F, which ſhall be the Perpendicular required.

SECT. 4.

To Divide a Line into 2 equal parts by a Perpendicular.

Let the given Line be A B, fix one foot of your Compaſſes in A, and ſetting them at any convenient diſtance above half the given line, deſcribe the Arches D and F, carry your Compaſſes to B with the ſame

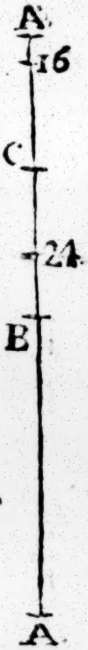


di-

distance describe the Arches C and E, so as they may cross the other Arches, and by those Intersections lay your Ruler, and draw the line GH, which will cut the given Line in the midst, and be a perpendicular also to it, which was required.

SECT. 5.

To Divide a Line given to any Proportion required, as to Divide the Line A *containing* 40 *Equal parts (let them be what they will) in proportion, as* 20 *is to* 30, *or as the Line* B *is to* C.

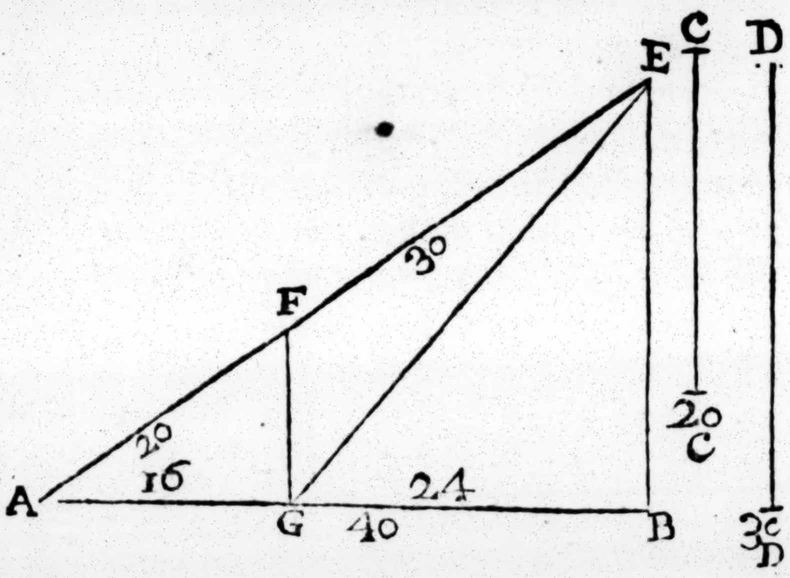


Add (the Lines B and C together) or 20 and 30, which makes 50, then say by the Rule of Proportion. If 50 the sum of both the given Terms give 40 the whole Line A, what shall 30 the greater Term give? Multiply and Divide, you shall have in the Quotient 24 for the greater part of the Line A, which taken from 40 the whole Line, there remains 16 for the other part.

For as 50 is to 40, so is 30 to 24, or 20 to 16, or as 30 is to 20, so is 24 to 16.

But to do this *Geometrically*, or by Lines, suppose AB to contain 40, and you would divide the same in such proportion as C to D.

First, From the point A, draw the Line A E at pleaſure, making the Angle EAB, then take in your Compaſſes the Line C, and ſet it from A to F, alſo take the Line D, and ſet it from F to E, and draw the Line E B perpendicular to AB, Then from the point F draw the Line F G parallel to EB, cutting the given Line A B in G, ſo is the Line A B divided as required.

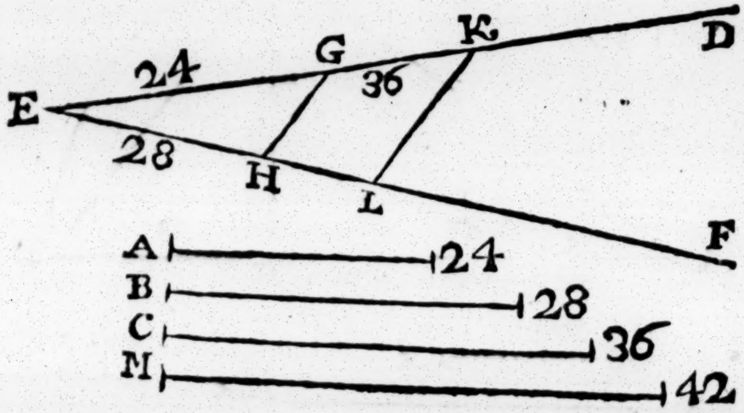


By dividing the Baſe of a Right Angled Triangle by this Problem, and drawing a Line from the oppoſite Angle to the point of Diviſion at G, you divide the Triangle it ſelf according to the ſame proportion.

SECT. 6.

Three Lines being given, to find a fourth in proportion to them. Or to perform the Rule of Three *in Lines, Draw 2 Lines, making any Angle as the Lines* E D *and* E F, *then the Three Lines given, being* A B C, *to find the Line* M.

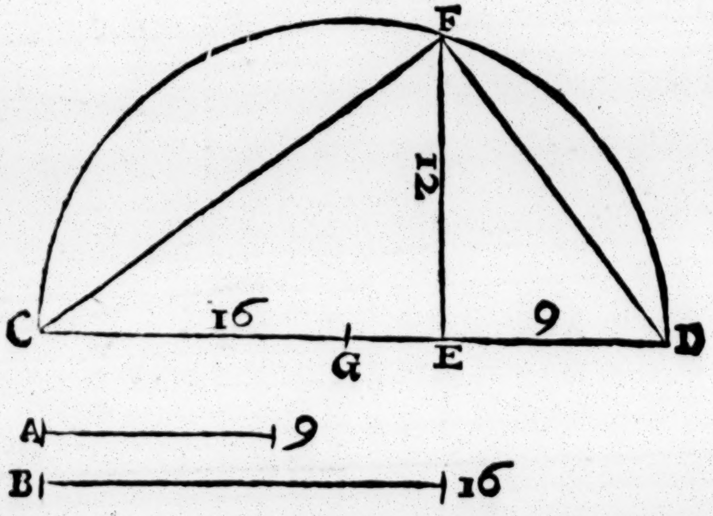
First take the Line A in your Compaſſes (that is 24 from a Scale of equal parts) and ſet it from E to G, then take the Line B 28. and ſet that length from E to H, then take the third given Line in your Compaſſes C. 36. and ſet it from E to K, and through the point K, draw the Line K. L parallell to G. H. ſo ſhall the Line E L be the fourth proportional required, For, as E G to E H, ſo is EK to EL.



SECT. 7.

To find a mean proportional Line between Two Lines given.

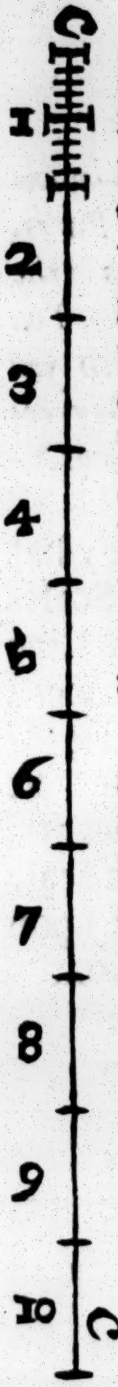
Let the two Lines given be A and B, which joyn together in the point E, making one right Line as CD. which divide into two equal parts in the point G, upon which point G with the diſtance GC or GD deſcribe the Semicircle CFD, then from the point E, (where the two Lines are joyned together) raiſe the perpendicular E F. cutting the Periphery of the Semicircle in F, ſo ſhall the Line EF be a mean proportional between the two given Lines, A and B. For, As
ED. 9. is to EF. 12. ſo is EF. 12. to CE. 16.



SECT. 8.

To Protract by Scale and Compass, or for want of a Scale to make a Line of equal Parts, which together with a Line of Chordes (the making whereof is taught Folio 67.) You may lay down any Line of what length you please: or any Angle of how many Degrees and Minutes soever, which take as follows.

First, For laying down any Line of what length you please. There are several Scales both of Brass and Wood, (that is) a Line of equal parts for protracting of Lines; and a Line of Chords for protracting of Angles; (together with other Lines at pleasure, as a Line of Rumbs and Secants, or the like:) But for want of such a Scale you may make these Lines your self upon Paper, which will serve upon occasion.



First then, To make a Line of equal Parts, open your Compasses to some small distance at pleasure, and run it ten times over. Then opening the Compasses to the Extent of the whole Ten small Divisions run that distance over nine times more, so have you a Scale of equal parts, and one of them divided into other Ten parts. Then taking 6 of these larger Divisions in your Compasses, make it the Radius or Semidiameter of a Circle, whereby you may draw a Line of Chords, or Rumbs, or Secants, *&c.* as you'l see hereafter.

Now to Protract by these, you shall be taught it in making those two following Geometrical Figures, *viz.* Squares and Triangles. To avoid unnecessary Repetitions, there are also *Protractors* (properly so called) by which you may lay down Angles, and avoid the drawing of many unnecessary Lines on the Paper; which said *Protractors* are made of a thin piece of Brass, cut into a perfect Semicircle, and divided into Degrees of a Circle, with a hole in the Center, and a Diameter drawn through the said Center, which is called the *Meridian Line.* The way of Protracting by this *Pro-*

Protractor, you ſhall alſo have exemplified in making of Squares and Triangles. And in the third place you may alſo Protract Angles from a Line of Right Sines as ſhall alſo be ſhewed you hereafter; and ſhall give you examples of each of theſe ſeveral wayes in every Figure, ſo you may make uſe of which pleaſes you beſt. But now to proceed to ſpeak of thoſe three famous Geometrical Figures, The Circle, Square, and Triangle particularly, and in order, and that in a double Capacity. Firſt, as they are Superficies, And, Secondly, as they are Solids. And firſt of a *Circle*.

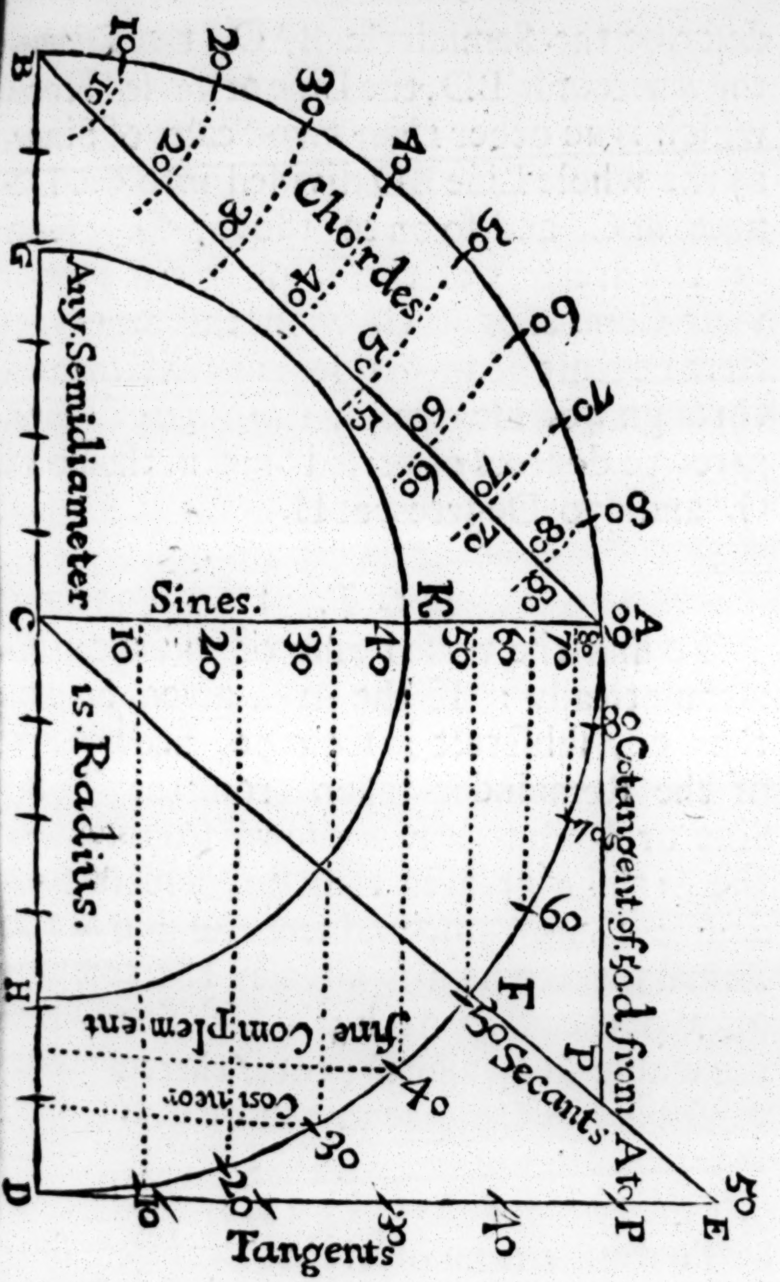
CHAP. II.

Of a Circle.

A *Circle* is a Geometrical Figure, which of all other is the moſt perfect, being an Emblem of Eternity, having neither Beginning nor End, and is made by the drawing a Line called the Circumference, equally every way diſtant from a Point in the middle called the Center, thorough which Point or Center a ſtraight line drawn from any part of the Circumference to another is called the Diameter; and any Line drawn

drawn from the ſaid Center to the Circumference is called the Semidiameter or Radius, or *Sinus Totus*, the whole Line of Sines, which being divided into 90 unequal parts are called Right Sines. The whole Circumference of the Circle, let it be bigger or leſs, is alwaies ſuppoſed to contain 360 Degrees, and is accordingly by Mathematicians alwaies divided into ſo many equal parts. The Semicircle into 180, and every Quadrant or fourth part of a Circle into 90. Each of theſe Degrees are ſubdivided into 60 Minutes, and each Minute ſuppoſed at leaſt to contain 60 Seconds, and every Second 60 Thirds, *&c.* as the largeneſs of the Circle will admit. Now by means of theſe Diviſions in the Circumference, for the finding out of Proportions, beſides the Line of Sines before-mentioned, there are many other Lines belonging to a Circle; as there are not only Right Sines, but alſo verſed Sines, Sines, and Sines complements, or Coſines; Tangents, and Cotangents, Secants, and Chords, for the underſtanding of all which, I ſhall give you this following Diagram, and how all theſe Lines are drawn and divided.

SECT.



SECT. 1.

First then, Setting one Foot of your Compasses in the Center at C, with the other describe

deſcribe the Semicircle BAD, the Diameter whereof is BD, the Line of verſed Sines; which is no other than two Scales of Sines, as the whole Line BD divided into 90. Degrees at C. and ſo on to 180. at D. Now for the drawing of theſe, place one foot of your Compaſſes at C, with the other deſcribe occult Semi-Circles: as GKH. drawn through 40. Degrees of the Line of Sines, gives 50 degrees of verſed Sines at the point G. and 130 Degrees at H.

SECT. 2.

To find the verſed Sine of any Arch Arithmetically: If the Arch given be leſs than 90, ſubſtract it from 90, and the ſine of the Remainder taken from the Total Sine or Radius leaves the verſed Line. If the Arch be greater than 90, ſubſtract 90 degrees therefrom, and ſeek the ſine of the Remainder, which is alwaies the complement of the given Arch; which ſine add to the whole ſine, and the total thereof is the verſed ſine of the given Arch deſired.

SECT. 2.

To draw a Line of Right Sines and Coſines through every Degree, or Tenth Degree of the Quadrant AD, draw right Lines parallel to the Line CD, till they cut the Lines AC, and thoſe are ſines: a Line drawn

per-

perpendicular from the ſame place to the line C D is the ſine complement or Coſine, as in the Diagram.

SECT. 4.

To draw a Line of Tangents and Co-tangents. For Tangents upon the point D erect a perpendicular DE, and through every degree of the Quadrant AD draw right Lines from the Center C, till they cut the perpendicular DE, and theſe are Tangents. Another perpendicular erected upon the point A, and touching the aforeſaid line drawn from the Center, are Co-tangents, as AP.

SECT. 5.

To draw a Secant is this very line drawn from the Center to E, which is the Secant of 50 degrees, the perpendicular it ſelf, which it cutteth, being the line of Tangents.

S *ECT.* 6.

To draw a line of Chordes, Draw a ſtrait line from B to A, and ſetting one Foot of your Compaſſes in B through every Degree, or every tenth Degree in BA, draw Arches to cut the ſtrait line, and that ſhall be a line of Chords.

All theſe lines are of admirable uſe in finding of proportions, ariſing from that Symmetrical

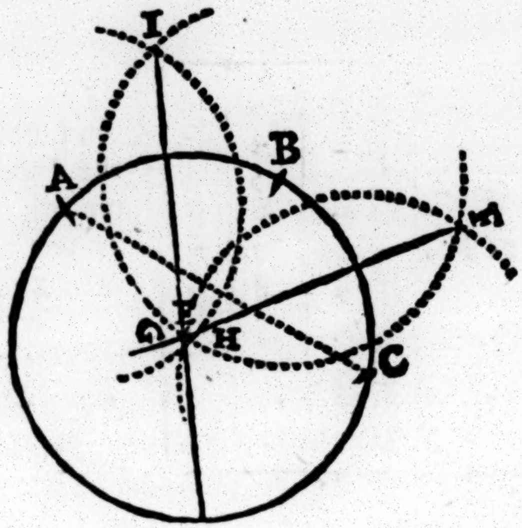
metrical proportion there is betwixt a Circle, and the parts and lines belonging thereunto; for according to the length of the Radius or Semidiameter, ſo is the Circumference; and ſo are the Sines, Tangents, Secants and Chords belonging to the ſame Circle or Radius: For Sines and Tangents you ſee what excellent Effects they have joyned with the line of Numbers in *Inſtrumental Arithmetick.* And for the Line of Secants, it is that which conſtitutes the Meridian Line, or Line of Latitudes of abſolute uſe in Navigation; and for the line of Chords, its excellency conſiſts in protracting of Angles, without which (excepting the Circle, which is perfectly round,) no other Geometrical Figure can be made or meaſured.

PROBL. I.

Three Points being given, not lying in a direct Line, to find the Center of a Circle, which ſhall cut all the 3 Points.

Let the Points be A B C, open your Compaſſes to ſome reaſonable Scantling more than half the diſtance betwixt A and B, with that wideneſs ſtrike a Portion of an Arch upon the point A; do ſo alſo upon the point B, and note the interſections of theſe

theſe 2 Arches, as at I and H, and produce a line from I by H infinitely. Then with your Compaſſes at the ſame diſtance as be-



fore, ſetting one foot in B; ſtrike an Arch towards C, and removing the ſame to C. ſtrike another Arch towards B, to croſs each the other, and where theſe interſect as at F and G, draw another ſtrait line to croſs the former; and where theſe two ſtrait lines interſect each other, as at E, is the Center of a Circle, which ſhall cut all the 3 points A.B.C, the thing required. The ſame Problem will alſo find the Center to any Segment of a Circle.

To

To Divide the Circumference of a Circle into any Number of equal parts. Divide 360 by the Number of Parts, the Quotient ſhews the Degrees as follows.

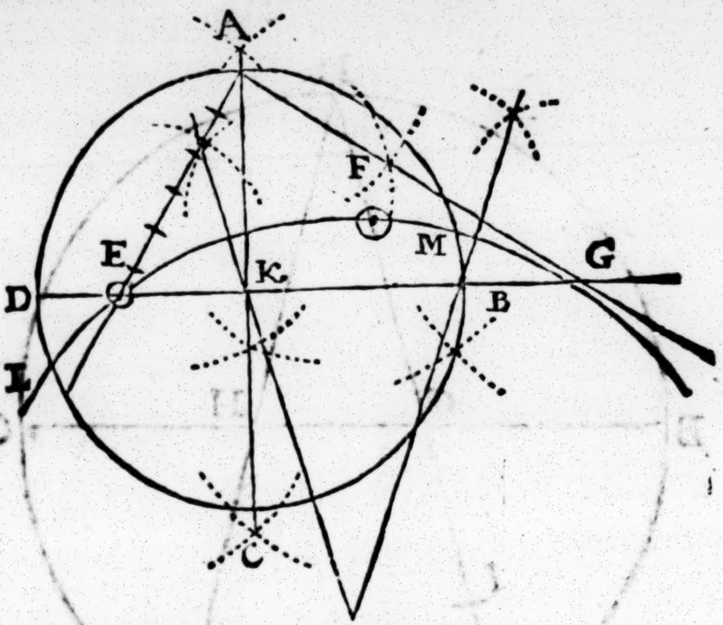
A Chord of		will divide the Circumference of a Circle into		
	120		3	Equal parts.
	90		4	
	72		5	
	60		6	
	51 26 M.		7	
	45		8	
	40		9	
	36		10	
	32 44		11	
	30		12	

And note that the Semidiameter will divide it into 6 parts, and the ſixth part of the Semidiameter into 36 equal parts, *&c.*

PROBL. 2.

Two Points within any Circle being given, how to deſcribe the Arch of another great Circle which ſhall paſs through thoſe two Points, and alſo divide the Circumference of the given Circle into two equal Parts.

Let the two Points be E, F, through either of which (as E) draw the right line D E, ſo as to paſs through the Center K: at right Angles draw the line AC, then draw the line EA, and from the point A erect a perpendicular AG, and ſo have you three points EFG, through which 3 points deſcribe

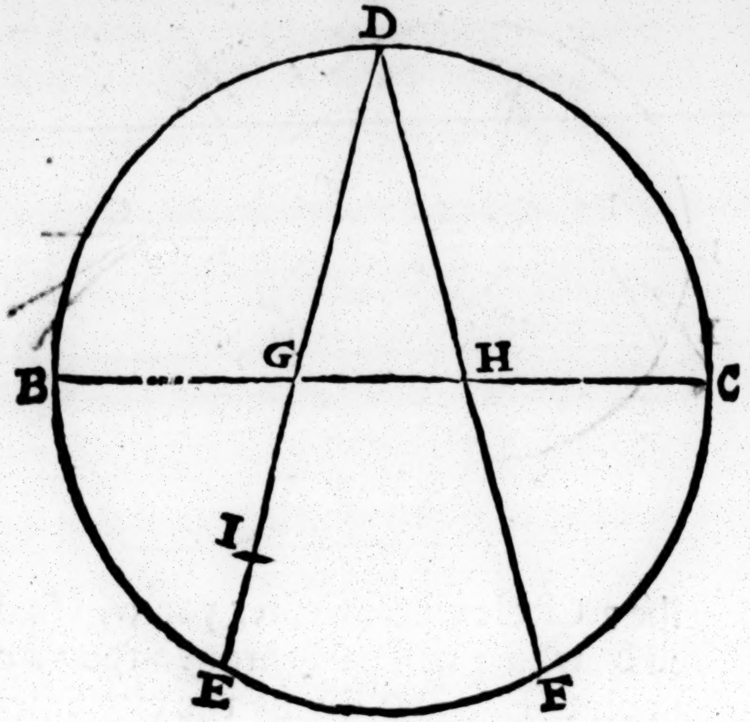


describe a Circle whose Center you will find to be H, which will pass through the two given points EF, and divide the circumference of the given Circle into 2 equal parts at L and M.

PROBL. 3.

To find a Right Line equal to the circumference of a Circle given.

Let the given Circle be BDCFE, Divide the upper Semicircle into halfes at D, and the lower Semicircle into three equal parts at F and E, and draw the lines DE, DF, which cut the Diameter of the Circle at G and H. Then make GI equal to GH. And the length DI is a very little more than the length of the quadrant BD, neither

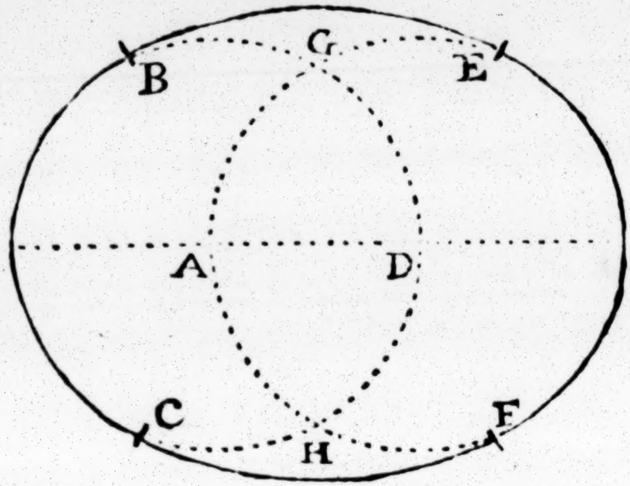


ther doth the excess amount unto one part of the Diameter BC, if it were divided into 5000, and four times the extent DI will be a little more than the whole circumference of the Circle.

PROBL. 4.

To draw a Geometrical Oval.

First, Draw two equal Circles, whose Centers are A and D, the Distance AD being equal to the Radius or Semidiameter of each Circle: Then open your Com



Compasses to the whole Diameter of the Circle, and set one Foot in H, with the other strike the Arch B G E, and with the Foot of the Compasses removed to G strike the Arch FHC, so will the Figure B C F E be a perfect Geometrical Oval.

CHAP. III

HAving shewn you how a Circle is produced, with all the Parts and Lines belonging thereunto; I come to shew you in the next place, How other Figures may be reduced thereunto; and afterwards shall shew you how to measure a Circle either superficial, or solid.

Now the Figures which may be reduced to a Circle, and measured by its Rules, are not many: My reading affords me only two material, which are the Oval and the Square.

And for the Oval divide the Distance on the line of Numbers between the length and the breadth of the Oval into two equal Parts, and the middle Point where the Compass stayeth on, shall be the Diameter of a Circle equal in Area to the Oval given.

SECT. 1.

To Reduce a Circle into a Geometrical Square, or by knowing the Superficial content of a Circle, to find the side of a Square equal to it.

Extract the Square Root of the whole content of the Circle, by taking half the Logarithme of the said content. As suppose the content to be 616, the Logarithme whereof is 2,789581.

Take the half thereof, which is the Square Root } 1.394790.

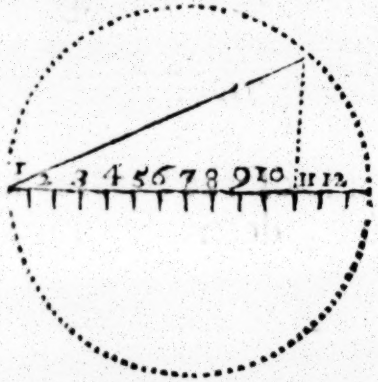
The nearst number answering hereto is 25, so the side of the Square ought to be near 25 of the said parts, whereof 616 were supposed to be the contents, whether Inches, Feet, Yards, &c.

SECT.

SECT. 2.

To Reduce a Circle into a Geometrical Square.

Having described a Circle, draw a Diameter, which divide into 14 parts, and on 11 of those parts erect a perpendicular, which extend to the circumference, and



from the point of intersection, draw a line to the extream part of the Diameter, which line shall be the side of the Square, containing the proportion (or Area) of the Circle.

SECT. 3.

To Reduce a Square to a Circle Instrumentally.

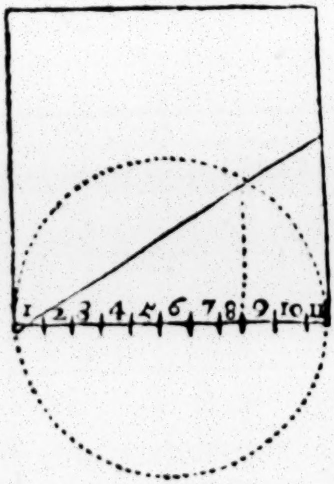
The side of the Square (as above) is the Square Root of a Number, which is the Su-

perficial content of a Circle equal to the ſaid Square, and knowing the ſuperficial content, you may find the Diameter. If the Diameter be 10, the Superficial content upon the line of Numbers is 78.54 parts. Divide the diſtance betwixt the given content, and 78.54 into two equal parts. The ſame Diſtance the ſame way will reach from 10 to the Diameter required.

SECT. 4.

To Reduce a Geometrical Square into a Circle.

Take any ſide of the Square, and Divide it into 11 parts, deſcribing a Semicircle to



the ſaid Diameter: Then on 8 of thoſe parts erect a perpendicular, which extend to

to the circumference of the Semicircle. Then draw a Line from the extream part of the Diameter, intersecting the perpendicular in the circumference of the Circle, and continue that line till it touch the side of the Square; so that line will be the Diameter of a Circle answerable to the Square given.

CHAP. IV.

TO conclude, (and to get out of this Circle) it remains only to shew you how to measure a Circle, and consequently an Oval, a Square, or any other Figure reducible thereunto.

First then for a Superficial or Flat Circle, such as may be a piece of Ground, or Board, or Glass perfectly round, or any other Flat, Round Superficies. Take these Rules.

SECT. I.

Having the Diameter, to find the Circumference.

Multiply the Diameter by 22, and Divide the Product by 7, as 28 by 22 is 616. Divided by 7 is 88.

SECT. 2.

Having the Circumference, to find the Diameter.

Multiply by 7, and divide by 22. As 88 multiplyed by 7, gives 616 divided by 22, is 28 the Diameter.

SECT. 3:

By the Diameter only to find the Area, or Superficial Content.

Multiply the Diameter by it ſelf, as 28 by 28, that makes 784. This Product multiplyed the ſecond time by 11, is 8624, which divide by 14, the Quotient will be 616, and that is the Area of the Circle.

For a half Circle, a Quadrant or any leſſer portion of a Circle, whoſe Point goeth to the Center. Half the Arch multiplyed by the Semidiameter, produceth the ſuperficial Content.

To meaſure the content of the Segment of a Circle, Take the chord $12\frac{1}{2}$, and the perpendicular 4, and multiply the whole of the one by two Thirds of the other, and it will come very near.

To find the Superficial content of a Circle, you need do no more but multiply half the circumference by half the Diameter. As the circumference being 44, the half

half is 22, the Diameter 14, the half is 7, Multiply 22 by 7, and the exact content of the Circle is 154.

To find the whole Diameter of a Circle, by knowing part thereof, and the length of the chord crossing the Diameter in that part: Suppose part of the diameter to be 4, the chord intersecting it $12\frac{2}{3}$, Square one half of the chord $6\frac{1}{3}$ multiplyed by it self is 40, divided by 4 the part of the Diameter given, rests 10, which added to the said part, shews the whole Diameter to be 14, whether the Section be of a Circle, or of a Globe.

CHAP. V.

Of a Solid Circle, *such as are* Globes, Sphears, Bullets, Balls, *or the like: To find their* Solid Contents.

MUltiply the diameter of the Circle by it self, as 28 by 28, makes 784, this product multiplyed the second time by the diameter 28, makes 21952, which two Multiplications are called cubing of the diameter. Then multiply that Cube Number by 11, makes 241472, and divide this

last product by 21, produceth the *Solid Content* $11498\frac{4}{21}$.

SECT. I.

To find the solid Content of a Globe another way.

Find first the Superficial content of a Globe, which to do, If you multiply half the diameter by half the circumference, the Product is the Superficial content of a Circle of equal diameter. As 28 multiplyed by 88 is 2464: And then if you Multiply the content of a Circle having the like diameter by 4, the Product is the Superficial content of a Globe, as 616 multiplyed by 4, produceth likewise 2464.

Now hereby to find the solid Content of a Globe.

Multiply the Superficial content by the sixth part of the Diameter, the Product shall be the solid content of the Sphere.

SECT. 2.

In like manner knowing the ſolid content of a Globe, to find the ſide of a Cube equal to the Globe.

Extract the Cube Root of the ſolid content of the Globe, which is done by taking the third part of the Logarithme of the ſaid ſolid content. And this Cube Root ſhall be the ſide of a Cube, which in ſolid contents ſhall be equal to the Globe given.

SECT. 3.

And to find the Superficial Content of the Segment of a Globe; ſay, If the whole Diameter 14 give the whole Superficial Content 616, what will 4 the part of the Diameter give, which you will find 176, and ſo of any other.

And thus much ſhall ſuffice to have ſpoken of a Circle, how it is produced, reduced, and meaſured.

Having hitherto treated of a Circle and Globe, I come now to ſhew you the Producing, Reducing, and Meaſuring of the other two principal Geometrical Figures, the *Square* and *Triangle*, as I have already done

concerning the Circle. And as the Circle was made up of a point, and a crooked or circular line drawn round about it; so *Squares* and *Triangles* are made up of *Lines* and *Angles*; *Squares*, and right lined *Triangles* of ſtreight *Lines* and *Angles*, *Sphærical Triangles* of crooked circular *Lines* and *Angles*: Now a ſtreight, or a right line is the neareſt diſtance drawn betwixt two points.

A circular or ſphærical Line is part of the circumference of a *Circle* bigger or leſs.

An *Angle* in a *Sphærical Triangle*, is (as was ſaid before) the diſtance of two Lines meeting in the *Angular* point, and meaſured in the circumference of a *Circle* at the diſtance of the *Radius*, or *Semidiameter* of the ſaid *Circle* from the ſaid Angular point, which pray obſerve very well.

Now every *Circle*, whether great or little, doth contain 360 degrees, every degree containing 60 Minutes, ſo that a Semicircle contains 180 degrees, a Quadrant or fourth part of a *Circle* 90 degrees. Now if an *Angle* be a Quadrant of a *Circle*, and contains 90 degrees, it is called a *Right Angle*: If it contains leſs than 90 degrees, it is called an *Acute Angle*, if more than 90 degrees, it is called an *Obtuſe Angle*. Theſe *Angles* and *Lines* do conſtitute all kinds of Geometrical Figures whatſoever.

CHAP.

CHAP. VI.

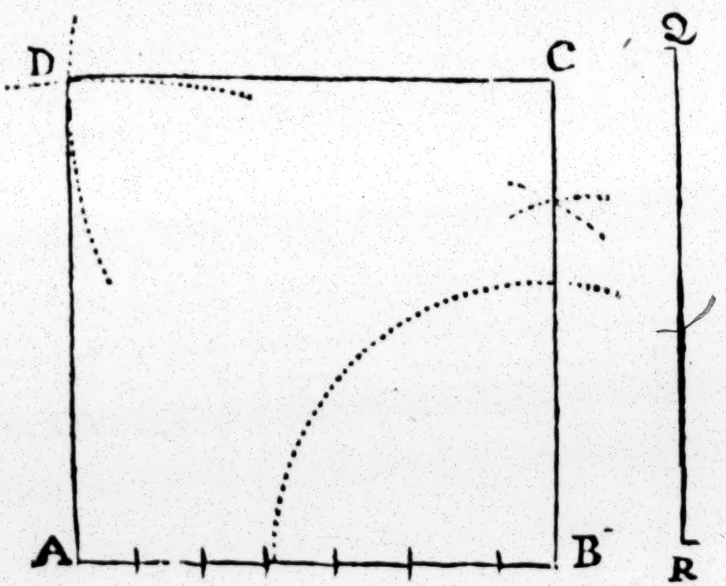
Of a Square.

THere are ſeveral ſorts of four ſided Figures, as the *Long Square*, or *Parallelogram*, whoſe Angles are all equal, a *Rhombus*, and a *Rhomboides*, whoſe oppoſite ſides are all equal ; and a *Trapezium* conſiſting of 4 unequal ſides, and 4 unequal Angles : But all theſe being reducible to 2 Triangles, or to a Geometrical Square , It is the Geometrical Square which I ſhall here ſpeak of, which conſiſts of 4 equal Streight Sides, and 4 equal Right Angles.

To make a Geometrical Square whoſe ſide ſhall be equal to a Line given, or to any Number of equal parts, ſuppoſing the ſame to be either Inches , Yards , Perches , or any other Meaſure.

As ſuppoſe you would make a Square, whoſe ſide ſhould contain 36 equal parts, (which ſuppoſe yards) or whoſe ſide ſhould be equal to the line Q R, which is the ſame. Firſt, From your line of equal parts, either upon your Scale, or prickt out upon Paper , take

take with your Compaſſes three of the larger diviſions, which ſignifie 30, then extend them further to 6 more of the ſmaller diviſions, which together make 36, and prick out, and draw a line of the ſame length as AB: Then by the *Probl.* fol. 54, erect a perpendicular upon the end of the ſame line at B, containing the ſame length. Then keeping your Compaſſes at the diſtance of QR, or AB, or BC, (all three being equal) ſet one Foot of your compaſſes in the point C, with the other



deſcribe the Arch at D: Alſo the compaſſes reſting at the ſame diſtance, place one foot in A, and with the other croſs the former prickt Arch in the point D. Laſtly, where

where theſe Arches interſect one another, make a prick with the point of your compaſſes, and from thence draw right lines from D to C, and from D to A, which ſhall include the Geometrical Square, ABCD.

Now if inſtead of erecting the perpendicular BC (as is before directed) you protract an Angle of 90 degrees, or a right Angle at the point B, it will effect the ſame thing, and a line drawn at the ſame Angle will be a true perpendicular.

Now I told you before, there were 3 ſeveral ways of protracting an Angle.

1. By Scale and Compaſs, or a line of Chords.

2. By a Protractor properly ſo called.

3. By a line of Right Sines, and I ſhall ſhew how to protract an Angle of 90 degrees at the point B, every one of theſe waies.

SECT. 1.

Firſt, By a line of Chords, and note that you muſt either have a line of Chords, whoſe Radius is ſhorter than the line AB, or DC, or elſe you muſt produce theſe lines till they meet with the Radius. Then with your Compaſſes ſet one foot at the beginning of your line of Chords, and the other extend

to

to 60 (where usually are brass Centers in most Scales) and with that extent setting one Foot in B, describe an Arch of a Circle, cutting the lines AB and CB, and then from the points where this Arch intersects the lines AB CB, measure with your compasses in the same Arch the Chord of 90 degrees, which is the distance from the beginning of the line of chords to 90 degrees or the whole line, and making a prick there, lay your Ruler to the said prick and the point B, drawing a line to the point B, and the Angle at B contained betwixt the two lines AB and B C shall be a Right Angle, or an Angle of 90 degrees, which was the thing required; and the like is to be done for any other Angle, of what quantity of degrees soever, as 30. 40. 50. 130. degrees, with their odd Minutes, if there be any; But I hope this Example may serve for all, or instead of many, so I shall mention no other in this place.

SECT. 2.

Secondly, By the *Protractor* do thus, put a Pin through the Center hole, fixing it in the point B, then lay the Meridian line of the Protractor upon the line A B, and by the edge of the Protractor make a mark against 90 degrees, and from that mark draw

draw a line to the point B, and the Angle included betwixt the two lines is an Angle of 90 degrees, as before. Suppose the Semi-circle BAD, (as in *pag*.65.) were a Protractor, the Limb thereof divided into degrees as there it is, the Point C. is the hole at the Center, the Line B D is the Meridian line, and at the Point A you find 90 degrees: The like is to be understood of any other Number of degrees whatsoever, more or less, or any other Protractor whatsoever. Lay the Meridian upon AB the line given, the Center upon B, and against 90 degrees, or the point A in the Protractor make a mark at C, and from C to B draw a Line, and AB, and CB will include an Angle of 90, degrees as required.

SECT. 3.

Thirdly, To Protract an Angle (which suppose 90 degrees) by the line of Right Sines, do thus. First, Set one foot of your Compasses in the beginning of the line of Sines, and extend the other to the Sine of 30 degrees, with this extent set one foot of your Compasses in the Angular point at B, and with the other strike an Arch of a Circle, cutting the line AB. produced to a sufficient distance. Then taking 45 degrees from the beginning of the line of Sines in your

your Compasses, which is half of 90. (or the half of any other Angle required ;) and setting one foot of your Compasses in the point of intersection where the Arch cutteth the Line AB, with the other foot of your Compasses make a mark in the said Arch, and from that mark draw a right line to the Angular point B. so is the Angle included double to the distance upon the Arch, *viz.* 90 degrees. And what is here said concerning the protracting of an Angle of 90 degrees, is in all Respects to be understood of any other Angle, or any Number of Degrees whatsoever, whether more or less, the same Rule of working being observed for one as well as the other.

And so much shall suffice to have spoken to the first particular concerning a Square, to wit, the Producing or Making of it.

CHAP.

CHAP. VII.

In the next place, I shall shew you the Reducing of it; or rather, the Reducing of other Figures to it, which take as follows.

SECT. I.

To Reduce a Long Square to a Geometrical Square.

WHich is no more but to find a mean proportional betwixt the length and the breadth of the Long Square, for that shall be the length of a side of a Geometrical Square equal in content thereunto. As Suppose the length of the long Square to be 64, and the breadth 16, between which two the mean proportional is 32; so that a Geometrical Square, whose sides are 32, will be equal to this long Square.

Now to find a mean proportional by *Common Arithmetick*, or *Arithmetically*, is taught before by Logarithmes, (*page* 33.) By the Lines of Numbers, Sines, and Tangents, (*page* 43.) By Protraction, (*page* 60.) where you have the variety of doing

doing it all theſe ſeveral ways, to which refer you.

SECT. 2.
To Reduce a Rhombus or Rhomboides to a Geometrical Square.

From one of the Obtuſe Angles let fall a perpendicular, and a mean proportional between the perpendicular and the ſide whereon it falls is the ſide of a Square equal to the ſame.

SECT. 3.
To Reduce a Triangle to a Geometrical Square.

Is to find a mean proportional between half the Baſe and the Perpendicular, which ſhall be the ſide of a Square equal to the Triangle: As ſuppoſe the Baſe 72, half whereof is 36; and ſuppoſe the perpendicular be 25, the mean proportional will be 30. And ſo any other irregular Polygon, (or many-ſided Figure) may be reduced into a Square, If firſt you Reduce the ſame into Triangles.

And thus much for Producing and Reducing a Geometrical Square: Now in the laſt place, To Meaſure it.

CHAP.

CHAP. VIII.

The last thing that remains to be spoke to about a Square, *is the Measuring of it; of which I come now to Entreat.*

SECT. 1.

To Measure the Square.

WHich is very easie and short, Multiply one of the sides in it self, or by it self, and the Product is the Superficial Content or Area: As 6 by 6 produceth 36, and 4 by 4 16, *&c.*

SECT. 2.

But besides this Superficial, or Flat Square, *there is also a* Solid Square; *such is a Cube, when the Top and the Bottom as well as the* 4 *sides are all* Squares *in fashion of a Dye.* Now to Measure this.

First, Multiply the length by the breadth, that gives the Superficial Content, and that Superficial Content multiplyed again by the Depth, gives the Solid Content in Cube Inches.

CHAP.

CHAP. IX.

Of a Triangle.

HAving ſpoken to the two former Geometrical Figures, the *Circle* and the *Square*. In the third and laſt place I come now to ſpeak of a *Triangle*, which is the moſt admirable of all the Three; The Producing of which the Reducing whereunto, and the Meaſuring whereof is the Sum and Subſtance of all the Mathematicks

For hereby we meaſure both Sea and Land, all Regular, and all irregular Bodies; the very Heavens themſelves, and all the Heavenly Bodies: All Real and Imaginary Magnitudes, Heights and Diſtances, whether acceſſible, or inacceſſible, as in the following part of this Diſcourſe you may beſt diſcern, when we come to particular Application of a *Triangle*, to *Surveying*, *Navigation* and *Aſtronomy*.

Triangles are of Two Sorts: Either 1. Plain: Or, 2. Sphærical, each conſiſting of three Sides and three Angles, which together make ſix ſeveral Parts: Any three of which Parts being given, any, or all of the

the other three may be found out by the Golden Rule, or Rule of Proportion.

For Sphærical Triangles, they properly belong to Aſtronomy, and the Reſolution of Aſtronomical Queſtions; therefore I ſhall paſs them by in this place, and refer the Reader till I come to ſpeak of Aſtronomy, and ſhall then ſhew you the Reſolution of them; and how thoſe Queſtions about which they are converſant, may be reſolv'd Inſtrumentally, with far more eaſe, more pleaſure, and more plainneſs. At preſent I ſhall ſpeak only to Plain or Right Lined *Triangles*, which are ſufficient for *Surveying* and *Navigation*; for taking Heights and Diſtances both by Sea and Land.

Now of theſe, If any one of the Angles be a Right Angle, or 90 Degrees, It is called a Right Angled *Plain Triangle*. If any of the Angles be an *Obtuſe Angle*, or more than 90 Degrees, it is called an *Obtuſe Angled Plain Triangle*. If all the Angles be Acute leſs than 90 Degrees (which they muſt be if they be neither of the former,) then it is called an *Acute Angled Plain Triangle*. And of theſe in my former Method as followeth.

CHAP.

CHAP. X.

1. IN the firſt place I ſhall therefore ſhew you, How to Produce, Protract, or Frame a Triangle.

2. I ſhall ſhew you how all other Figures, whether Regular or Irregular may be Reduced to one Triangle, or however to many, and ſo meaſured by its Rules.

3. How to Meaſure a Triangle, or to find the Contents either of a Superficial, or of a Solid Triangle. And likewiſe by ſome of the ſix parts given, (*viz.* Sides and Angles.) To find the reſt, which is properly called the Reſolution of a Triangle, or Trigonometry.

Firſt then, To Produce, Protract, Lay down, or Frame a Triangle, is no more but the protracting or laying down of Lines and Angles. The Lines or Sides are laid down from a Line of Lines, or equal Parts, and the Angles are protracted by a Line of Chords, or by a Line of Sines, or a Protractor, as you ſee is taught in the making of a Square, *pag.* 83, 84, 85, *&c.* But becauſe there is only mention made of a Right Angle, (a Square admitting no other) I ſhall here

here give you Examples of other Angles alſo. Now you muſt know that every Acute-angled Triangle, whoſe Angles are all Acute, doth contain 180 degrees, all the Angles added together; A Right Angled Triangle, beſides the Right Angle, which is alwaies *90* degrees, contains other *90* in the two other Angles; the one whereof is alwaies the complement of the other to 90 degrees more, being in the whole 180 degrees, or a Semicircle exactly. An Obtuſe Angled Triangle, by how much the more the Obtuſe Angle is more than 90, by ſo much the other two are leſs than *90*: All three together notwithſtanding making exactly 180 degrees as the former.

So that the Three Angles of every Triangle are equal to two Right Angles, or 180 degrees, whether it be an Acute, Obtuſe, or Right Angled Triangle.

As the Angles are thus diſtinguiſhed into Right, Obtuſe and Acute, ſo alſo are the Sides of a Right Angled Triangle (which is of moſt uſe and excellency) diſtinguiſhed into theſe three, The Baſe, the Perpendicular, and the Hypothenuſe. (In other Triangles, either Obtuſe, or Acute, they are called the Hypothenuſe, and the 2 Catheti.) Or others divide them into the ſubtending ſide, which is the ſide oppoſite to any Angle, and the two containing ſides which

which are the sides next on either side the said Angle.

In a Right Angled Triangle, the Hypothenuse is alwaies the side opposite to the Right Angle, the Base is usually accounted the longer of the other two, and the perpendicular the shorter, including the Right Angle betwixt them.

Now to make a Triangle, whose Base is 400 inches, feet, or yards, and perpendicular 231, and Hypothenuse 462; the Right Angle 90 degrees, the Angle at the Base 30 degrees, and consequently the other Angle at the top of the perpendicular 60 degrees, being the complement of the other Angle at the Base to 90 degrees. Take this following Example,

First, Draw a line at pleasure A B, and with your Compasses take 4 of the larger equal parts from your Scale or Line of equal parts, and mark that distance upon the said Line AB, signifying 400, then upon the Point B raise a Perpendicular or protract an Angle of 90 degrees (which is all one) as is taught *page* 86. either by a Line of Chords, a Line of Sines, or Protractor. Then with your Compasses take 231 from the Line of equal Parts for the length of the perpendicular; and setting one Foot in the Angle at B, make a mark with the other for the length of the perpendicular: Then upo

upon the Angle A, at the Base protract an Angle of 30 degrees on this manner.

SECT. 1.

First, by a Line of Chords, set one foot of your Compasses at the beginning of the Line, and extend the other to 60 Degrees for the Radius: Then with this extent setting one foot in the point A, describe the Arch DE, cutting the Base in E, and then setting one foot of your Compasses again in the beginning of the Line of Chords; extend the other to 30 degrees, (that being the number of degrees the Angle is to contain) and setting one foot of the Compasses in the point E, where the Arch does intersect the Base; with this extent of 30 degrees make a mark, then laying your Ruler to the said mark, and the Angular point A, draw a line at length, or till it meet with the perpendicular, so shall the Angle at the Base contain 30 degrees, and the other at the Perpendicular 60, being the complement to 90 degrees.

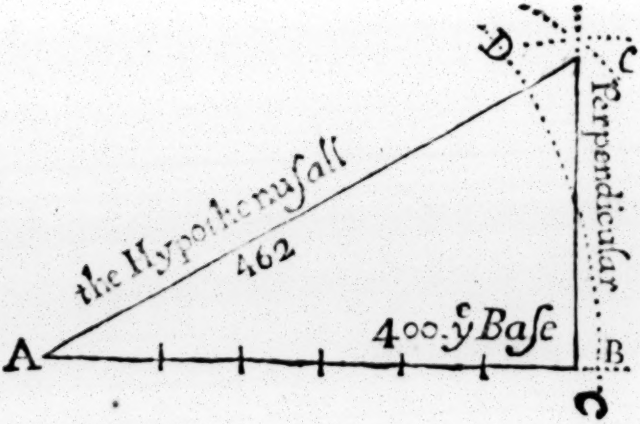
SECT. 2

Now by the Line of Sines to protract the said Angle of 30 degrees. First, take in your Compasses 30 degrees of the line of

ſines, and at that diſtance draw the Arch, and at the diſtance of 15 degrees (which is half of 30 the Angle required,) draw the Hypothenuſal, ſo ſhall it be an Angle of 30 degrees.

SECT. 3.

And laſtly, To protract this Angle of 30 degrees by the Protractor, do thus: Prick the Pin through the Center hole in the point A, and lay the Meridian line upon the Baſe AB, and making a mark by the edge of the *Protractor* againſt 30 degrees, by the ſame mark, and the point A, draw the Hypothenuſal AC.

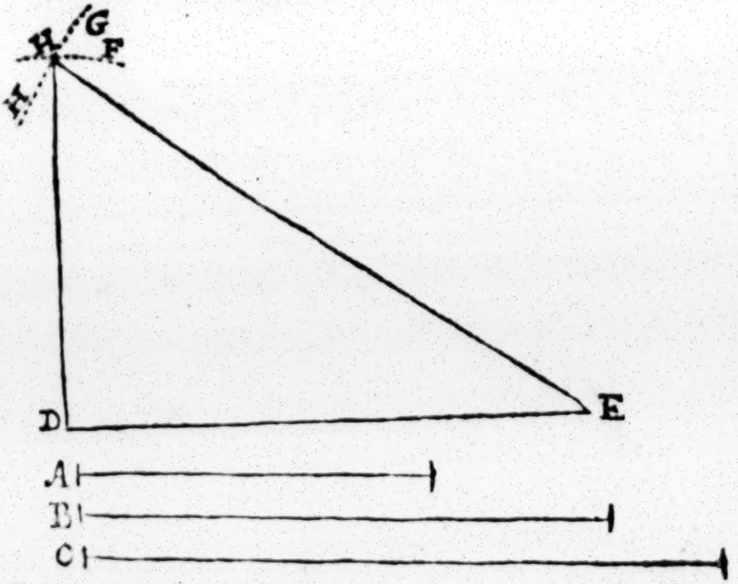


So that by any of theſe three ways you may protract an Angle, and any other Angle as well as this in the Example; obſerving the ſame Rules, which together with the

the ſides protracted from a line of equal parts, makes up the compleat Triangle, ABC; and by conſequence any other Triangle whatſoever: Only in an obtuſe Angle, where it is more than 90 degrees, as ſuppoſe 140, you muſt firſt prick out 90 degrees, and from thence the remainder that is 50 degrees more, which together make 140: And the ſame Rule is to be obſerved for odd Minutes, as is here given for degrees all along.

SECT. 4.

Three Lines being given, ſo that the two ſhorteſt together be longer than the Third, to make thereof a Triangle.



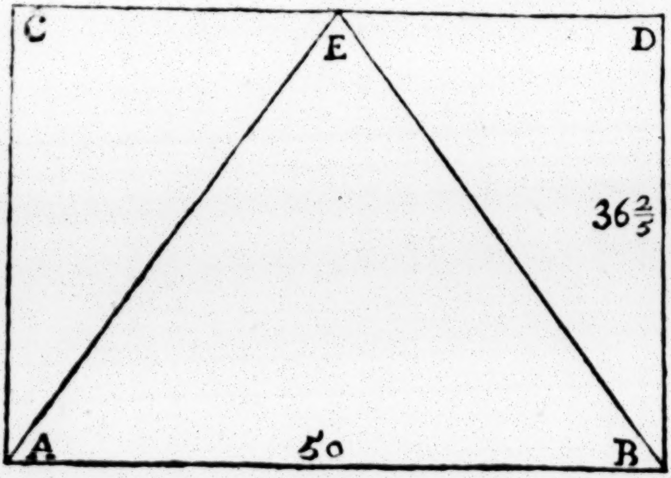
As ſuppoſe the Lines given be A, B, C, Firſt draw the line DE equal to the line B, then take with your Compaſſes the line C,

and ſetting one foot in E, with the other deſcribe the Arch HG: Alſo taking the given line A in your Compaſſes, and placing one foot in D, with the other deſcribe the Arch HF, cutting the former Arch HG in the point H. Laſtly, if from the point H you draw the lines HE and HD, you ſhall conſtitute the Triangle HDE, whoſe ſides ſhall be equal to the three given lines, ABC.

SECT. 5.

To make a Triangle to contain 910 *equal parts, (ſuppoſe Perches) whoſe Baſe ſhall be* 50 *of the ſame equal parts, be they what they will.*

Double the number of Perches or equal Parts 910, and they make 1820; divide this



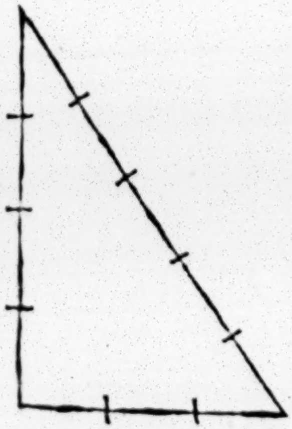
by the Baſe 50, the Quotient will be the length of the perpendicular of the Triangle,

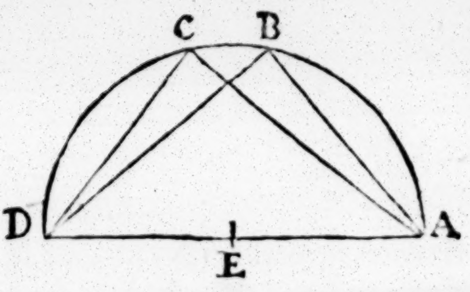
gle, to wit, $36\frac{2}{5}$. Then from any Scale of equal parts lay down AB the Base 50, then upon B raise the Perpendicular BD. $36\frac{2}{5}$; and draw the line CD parallel to AB: Then from any point in the line CD (as from E) draw the lines EA and EB, including the Triangle EAB, which shall contain 910 equal parts as required. And the like, if you would make a Triangle to contain any other number of Perches or equal Parts, the Rule is the same *mutatis mutandis*.

SECT. 6.

To make a Right Angle readily.

Take a Line for the Hypothenusal, and divide it into five equal parts, then take





three of those equal parts for the Base, and four for the Perpendicular, and joyn these

 toge-

together: Or, draw a Semicircle, as ABCD upon the Center E, (D A the Diameter) and from A draw a line to the Circumference, as at B or C: Then from B or C draw lines a to D, so have you made a right Angle, as ABD, or ACD.

SECT. 7.

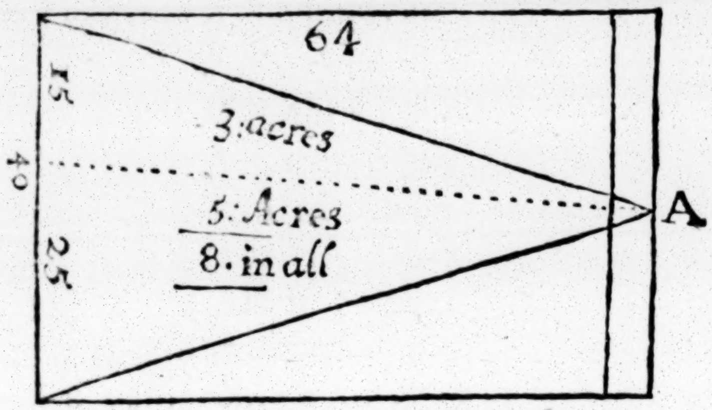
To Divide a Triangle into any proportion whatsoever.

By the Fifth *Geometrical Problem*, (*p.* 58.) Divide the Base of the said Triangle in proportion, as 20 is to 30, as there it is done, and from the point of Division at G draw a line to the opposite Angle at E, so is the Triangle it self divided in the same proportion; for as the line C 20 is to the line D 30, so is the Triangle EAG to the Triangle EBG.

To do this Arithmetically.

Suppose the whole Triangle to contain 8 Acres, and if it be required to Divide the same into 2 parts by a line drawn from the Angle A, the one part to contain 5 Acres, the other three. First measure the whole length of the Base, which suppose 40, then say by the Rule of Proportion. If eight Acres (the

(the Quantity of the whole Triangle) give 40 (the whole Base) what parts of the



Base shall five Acres give? Multiply and Divide, the Quotient will be 25 for the greater Segment, which being deducted from 40 (the whole Base) there will remain 15 for the lesser Segment, then draw a line from the opposite Angle, which shall Divide the Triangle according to the proportion required.

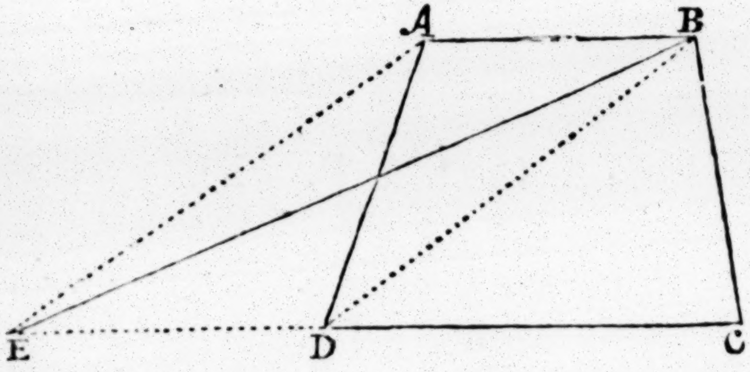
CHAP. XI.

THus much shall suffice for producing, and framing a Triangle, and also for dividing the same: In the next place according to my former Method, to shew how other Figures may be Reduced to Triangles:

gles: And herein appears the excellency of a Triangle above the rest, that all Geometrical Figures, whether Regular or Irregular, which can be reduced to neither of the former, that is, neither to a Circle, nor a Square; may yet be infallibly reduced to a Triangle, or however a certain Number of Triangles. *Leyburn* in his First Book of his compleat Surveyor containing *Geometrical Problems*, shews *Probl.* 30 31. 32 33. How to Reduce a *Trapezium*, or any irregular Plot of 5, 6, 7, or 8 sides into a single Triangle, which I abridge in this manner.

To Reduce a Trapezium into a Triangle containing 4 unequal sides.

Extend the line DC, and draw the Diagonal B. *D* Then from the Point A draw

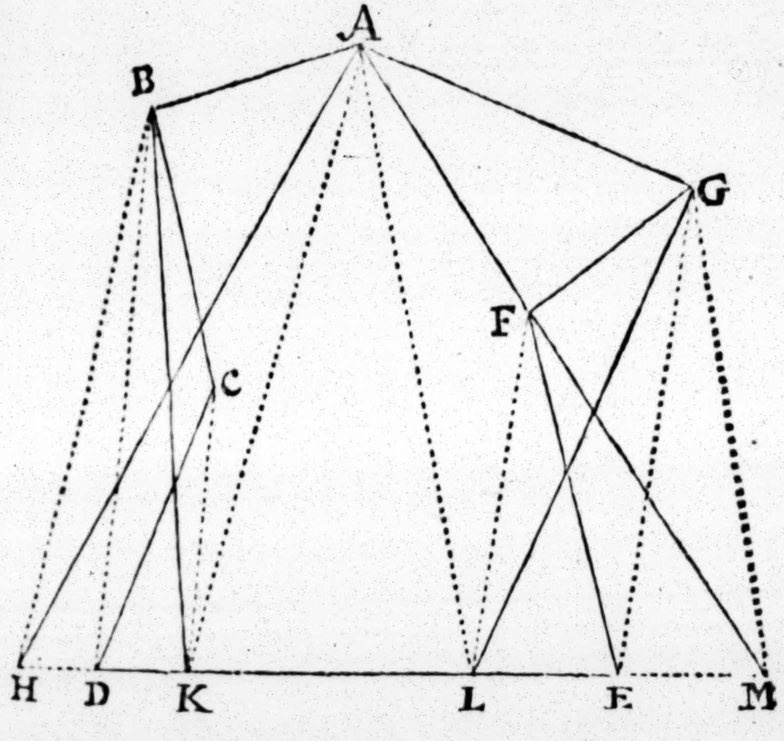


the line AE parallel to BD, extending it till it cut the side CD in the point E. Lastly

From

From B draw the line BE, conſtituting the Triangle EBC, which ſhall be equal to the Trapezium, ABCD.

How to Reduce an Irregular Plot of 5. 6. 7. *Sides into a Triangle.*



Let ABCDEFG. Firſt Reduce this Figure of 7 ſides into a Figure of 5 ſides, keeping the ſame quantity, ABKLG on this manner. Draw the line BD, and parallel thereunto CK, then draw the line BK, whereunto the two lines BC and CD are thereby reduced. Then draw the line GE, and parallel thereunto FL, then if you draw the line GL,

F 5 the

the two ſides, GF and FE ſhall thereby be reduced to one ſtraight line, *viz.* GL, and the whole Plot to a Figure of five ſides ABKLG.

Now to Reduce this to a Triangle.

First, Produce the ſide DE on both ſides at pleaſure, then draw the lines AK and AL, and parallel to them the lines BH and GM, cutting the line D E, being extended in H and M. Laſtly, If you draw the lines AH, A M, you ſhall conſtitute a Triangle AHM equal to the Irregular Plot ABCD EFG.

And having ſhewn you to Reduce theſe Figures into a ſingle Triangle, I ſhall alſo ſhew you how to Reduce the like Figures into ſeveral Triangles, which being ſeverally meaſured and added together, give the content of the whole Plot or Figure.

Now for the doing hereof obſerve this general Rule, That the Number of Triangles contained in any Figure or Plot, will be alwaies two fewer, or not ſo many by Two as the number of Angles, or Sides which the ſaid Figure or Plot doth contain.

As any Square or Trapezium which contains 4 Angles, and 4 Sides may be eaſily reduced to 2 Triangles drawn from Corner to Corner, ſo likewiſe a Plot of 5. 6. 7. or 8 ſides

8 sides may be easily reduced to 3. 4. 5. or 6 Triangles, as will appear upon practice, and needs no *Example*. Now these Triangles measured particularly, and then the whole added together, gives the content of any Irregular Plot or Figure whatsoever.

CHAP. XII.

THerefore in the third place to shew you How to measure a Triangle, and by consequence any other Plot or Figure, take this Rule, that half the perpendicular and the whole Base, or half the Base, and the whole perpendicular multiplyed the one by the other, gives the Area or Superficial Content of the said Triangle.

SECT. 1.

Now the perpendicular of a Triangle is a Plumb line let fall from any Angle upon its opposite side, which we call the Base: How to let fall this perpendicular from any Angle or Point you have *Probl.* 3. *pag.* 55. to which I refer you.

Now for an Equilateral Triangle, or a Triangle having three equal sides, observe this

this Rule: That the power of the ſide is to the power of the perpendicular let fall from any of the Angles to the ſubtendent ſide in proportion ſeſquitertia, or as 4 to 3, or $\frac{3}{4}$ of the other. Note the Power of a Line is the Square thereof.

SECT. 2.

Any three ſides of any Triangle being known, to find the Perpendicular: The greateſt ſide being aſſigned for the Baſe, upon which the perpendicular ſhall be ſuppoſed to fall: Firſt find the Sum of the other two ſides, (which ſuppoſe 13 and 11.) Then obſerve the difference betwixt theſe two ſides, which is 2; that done, ſuppoſe the whole Baſe to be 20: Say by the Rule of Proportion, As the whole Baſe 20 is to the Sum of the other two ſides added together, *viz.* 24, ſo is the difference of the other two ſides, *viz.* 2. to a fourth number $2\frac{2}{5}$, which being deducted out of the Baſe, the perpendicular will fall in the middle of that which remains, $2\frac{4}{10}$ deducted from 20, reſts $17\frac{6}{10}$, the half whereof is $8\frac{8}{10}$: a line drawn from that point from the Baſe to the oppoſite Angle is the true perpendicular to the ſaid Triangle.

Ano-

Another way to find the Perpendicular of an Equilateral Triangle.

Multiply one of the ſides by 13, and divide the Product by 15, the Quotient is the perpendicular, ſaies *Brown* in his Triangular Quadrant.

CHAP. XIII.

THus much for meaſuring a Superficial Triangle, now a ſolid Triangle is that Figure, or Solid Regular Platonick body, called a *Tetrahedron*, contained under 4 equal and Equilateral Triangles, or a Triangular Pyramid: The ſolid content whereof is found by Multiplying the Area or Superficial content of the Baſe by one third part of the length of the Pyramid or Tetrahedron from the midſt of one plain to the Apex or Top of the Solid Angle oppoſite thereunto.

Note that this Triangular Pyramid is little more than [illegible] of a Cube of equal Baſe and Altitude: A Cylinder is [illegible] of a Cube of equal Baſe and Altitude.

A Globe is [illegible] of a Cube, or [illegible] of a Cylinder, whoſe ſides and diameters are equal.

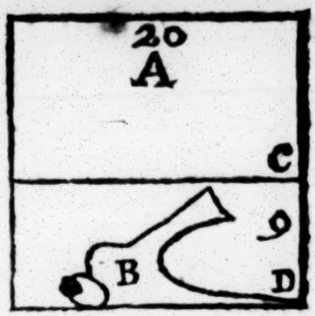
How

How to Measure the Solid Content of any Body, how irregular soever it be, The Form and Fashion not regarded.

To perform this, you must prepare an hollow Cube, into which put your Irregular Body, which being placed therein, you shall pour in so much Water, till it no more than just cover the Body in the Cube, then make a mark in the inside of the Cube where the Superficies of the Water toucheth. This done, take but the irregular Body, and mark again directly under the former, where the Brim of the Water now touches; for the distance of these two marks multiplyed by the Square of the Cubes side, produceth the Crassitude of that Irregular Body. For Example.

Suppose

Suppoſe A to be the Cubical Hollow Veſſel, whoſe inward ſide ſuppoſe to be 20 inches, B the irregular Body, whoſe Solid Content is required. Firſt then I put B into the Hollow Cube A, and pouring in water till it be covered, admit the top of the water to reach to C: Then taking out the irregular Body again, admit the



$$\begin{array}{r} 20 \\ 20 \\ \hline 400 \\ 9 \\ \hline 3600 \end{array}\ \text{Inches.}$$

Superficies of the Water fall to D, then meaſure the diſtance betwixt C and D, ſuppoſe 9 inches, which Multiplyed into 400, the Square of the Cubes ſide produceth 3600; and ſo many Cubical Inches are contained in the Irregular Body B. By this you may meaſure any Part or Portion of a Solid Body.

Now having ſpoken of a Solid Circle, *viz.* a Globe, and of a Solid Square, *viz.* a Cube: And now of a Solid Triangle, *viz.* a *Tetrahedron*. Theſe two laſt being two of the five Platonick Bodies, I ſhall add the other three.

An *Octaedron* is contained under eight equal and equilateral Triangles.

An

An *Icosaedron* is contained under 20 equal and equilateral Triangles.

A *Dodecaedron* is contained under 12 equal Equilateral and Equiangular *Pentagons.*

A *Polygon* is any Figure of many ſides, and a Regular *Polygon* is one whoſe ſides are all equal. Now to meaſure any Regular *Polygon.* The diſtance from the Center to the middle of one ſide, and the half Sum of the meaſure of all the ſides multiplyed together, ſhall be the true Area or Content thereof. To meaſure any Irregular *Polygon,* ſee *pag.* 105, by Reducing it to a Triangle.

If the Area or Content be 100, *then the ſides of theſe Regular Figures are as follows.*

An Equilateral Triangle, { its perpendicular		13	1[illegible]
{ its ſide		15	2
A Square its ſide.		10	
A Pentagon, which is a Figure of 5 ſides.		7	62
An Hexagon of 6 ſides, its ſide		6	2
An Heptagon of 7 ſides, its ſide.		5	2[illegible]
An Octagon of 8 ſides, its ſide.		4	55
A Nonagon of 9 ſides.		4	[illegible]
A Decagon of 10 ſides.	3 6	3	64
The Semidiameter or Radius of a Circle.		5	64

A Cone differs from a Pyramid, becauſe a Cone hath alwaies a round Baſe and Superficies like a Sugar-loaf: But a Pyramid hath an Angular Baſe and Superficies of ſe-

veral

veral ſides: The way to meaſure both as to the Solid Content. Firſt, find the Superficial content of the Baſe, and Multiply the ſame by the third part of a perpendicular from the ſharp end to the Baſe. And according to theſe Proportions beforementioned is any other Quantity or content to be found out: As ſuppoſe I would have a Triangle to contain 200, what muſt the ſides be? The half diſtance in the line of numbers betwixt 100 and 200 will reach from 15.2 to 21.5 for its ſide, and from 13.123 the perpendicular of a Triangle, whoſe Area is 100; the ſame extent will reach to 18.6 the perpendicular of an Equilateral Triangle, whoſe Area is 200, &c.

SECT. 3.

The three ſides of a right-lin'd Triangle being given, to find the Superficial Content.

Add the three ſides together, (as 20. 13. 11.) which make 44, the half of which is 22; from which half Sum ſubſtract each ſide ſeverally to the end you may have the difference betwixt that half Sum and each ſide, which will be 2. 9. 11. Then add the Logarithmes of the ſaid half Sum and of thoſe differences together, and laſtly, Divide the Sum of all thoſe Logarithmes

garithmes by 2, so you will have the Lo-
rithme of the Superficial Content or A
of the Triangle.

The half Sum	22.	1 3424
The Differences	2.	0.3010
	9.	0.9542
	11.	1.0413
The Sum of the Logarithmes —		3.63908
The Area or Content req.	66.	1.81954

OF

S

OF GAUGING ALL SORTS OF VESSELS.

GAuging of Veſſels is no more but Meaſuring of Solids. Theſe Solids are to be Reduced to ſome Regular Geometrical Figure (as is taught before when we treated of Meaſuring of Solids in General) eſpecially to one of theſe Three.

1. The Solid Square or Cube.
2. The Solid Triangle, or any Irregular Solid.
3. The Solid Circle or Cilender.

1. *To*

1. *To Measure a Solid Square, or any Vessel in the form of a Cube.*

The Rule is this

Multiply the Length by the Breadth, and that Product by the Depth (in Inches) and that gives the contents in Square Inches.

Now to bring these Inches into Ale, Wine, or Corn Gallons.

Obſerve this Rule once for all, That formerly the Ale Gallon was accounted to contain 288 Cube Inches. But by the care and pains of Mr. *Nicholas Gunton*, the juſt quantity of the Quart for Ale remaining in the hands of the Chamberlains of His Majeſties Exchequer appears to be 70½ Square Inches, which makes the Gallon 282, as ſome have of late found to their no ſmall coſt. And I am of opinion (ſaith *Mayne*) that if the Wine Gallon were carefully examined, (and ſo alſo the Corn Gallon) they would prove to contain leſs than they are commonly holden to do.

But to proceed, for Ale divide the Product in Square Inches by 282; for Wine by 231, and for Corn by 272¼, and the Quotient of each is the Gallons; and if any

any Fractions remain to Reduce it into Pints, multiply the Numerator by 8, (ſuppoſe $\frac{128}{282}$ be the Fraction) that is 128 by 8, and Divide the Product by the old Denominator 282, the Quotient ſhews the Pints. Thus much for meaſuring the Square or Cubical Veſſel, you may alſo find *pag.* 89, 90. what other figures may be reduced to this, as a Long Square, a Rhombus, or Rhomboides, a Triangle, a Circle, with its Segments, *&c.*

2. *Now in the ſecond place to Meaſure a Solid Triangle, or any Veſſel of a Triangular Form.*

The Rule is,

Multiply the perpendicular by half the Baſe, or the whole Baſe by half the perpendicular, and divide the Product by 282, the Quotient will be the Ale Gallons contained in one Inch of Depth upon that Triangle. If the Veſſel be of an Irregular Form, Divide it into ſeveral Triangles, and let fall perpendiculars in each, and ſo find their ſeveral Areas and Contents, and then add them together: So have you the Content or Area of the whole Figure, be it never ſo Irregular.

3. *In*

3. *In the third place to measure any Vessel which is of a Circular or Cylindrical Form; That is to say, Either Cylinders, Or, To be Reduced to Cylinders.*

The Rule is this,

Multiply half the Diameter by half the Circumference, gives the Area or Superficial Content of the Base or Bottoms, and then Multiply that Product by the Depth, gives the Solid Content in Inches, which Square Inches you may bring into Gallons, and Pints, as before is taught.

To find the Diameter of any Circle by the Circumference.

Multiply the Circumference by 7, and Divide the Product by 22.

To find the Circumference by the Diameter.

Multiply the Diameter by 22, and Divide the Product by 7.

To find the Area or Superficial Content of a Circle by the Diameter.

Square the Diameter, that is, Multiply it by it self, then Multiply the Product by

by 11, and divide by 14, the Quotient gives the Area or Superficial Content: For as 14 is to 11, ſo is the Square of the Diameter to the Area, or at leaſt the neareſt Approximation to it for Common Practice, or which comes infinitely near the Truth. As Unity is to 3.14159, ſo is the Square of the Semidiameter to the Area in Square Inches. Or,

Take this ſhort Rule to find the Ale or Wine Gallons in any Circle by the Diameter, and one Inch in depth.

Square the Diameter, (that is, Multiply it by it ſelf,) and divide that Square or Product by 359 for Ale Gallons, or by 294 for Wine Gallons; and then Multiply that Area or Superficial Content by the Length or Depth of the Veſſel, and that will give you the Solid Content of the whole Veſſel in Gallons.

Thus far we have ſuppoſed the Veſſel to be meaſured to be perfectly Circular or Cylindrical; and the Top and the Bottom to be both of the ſame Diameter: But if the Top and the Bottom be of different Diameters, (or which is the ſame) the Head and the Bung, then you are to find a Mean Diameter,

Which

Which when the Vessel is a perfect Cone, and the sides strait.

First, Find the Area of the Base or Bottom, and multiply that Area by the third part of the Altitude, the Product is the solidity of the whole Cone.

If the Vessel be not a whole Cone, but only a Part or Frustum of a Cone.

Add the Diameters at Top and Bottom together, and take the half of that Sum for the mean Diameter.

To Measure a Vessel, which is an Irregular Cylinder, (such as a But, Pipe, Hogshead, Barrel, &c.) find the Mean Diameter by these Rules.

1. Add double the Bung Diameter to once the Head Diameter, and Divide their Sum by 3, the Quotient take for your mean Diameter.

2. More exactly, To the doubled Square of the Bung Diameter, add the Square of the head Diameter, and that Sum Multiply by the Cask length, the last Product Divide by 1077, the Quotient is the Ale Gallons; or by 882 Quotes the Wine Gallons contained in that Cask.

3. *By*

3. *By Mr.* Oughtred's *way of Measuring the Frustum of a Spheroid.*

First, take the Diameter at the Bung, and find the Area of the Circle answerable thereunto, and take two thirds of that Area. Secondly, Take the Diameter at the Head, and find the Area of that Circle, and take one third of that Area; add these two Sums together, and multiply the whole by the length of the Vessel, the Product gives the content of the whole Vessel in Cubique Inches, which divided by 282 for Ale, or 231 for Wine, gives the Gallons.

To Measure a Vessel, which is an Irregular Cylinder, such as a But, Pipe, Hogshead, Barrel, or the like.

Take the Diameter at the Bung, which suppose 23 Inches, and then take the Diameter at the Head, which suppose to be 20, the difference is 3: Now (in respect to the bending of the Staves in proportion, as 7 to 10) Multiply this difference 3 by 7 makes 21, and divide the Product by 10, makes $2\frac{1}{10}$, which added to the lesser Diameter 20, makes $22\frac{1}{10}$ for the Mean Diameter: Half of this Mean Diameter multiplyed by half of its proportional Circum-

ference, gives the Area or Superficial Content, and that multiplyed by the Length of the Veſſel, gives its Solid Content in Inches, and thoſe Inches reduced to Gallons by dividing by 282 for Ale, and 231 for Wine, *prout ſupra.*

To find a Mean Diameter by Gunters *Line of Numbers.*

Extend the Compaſſes from the Gauge Point to the mean diameter, the ſame Extent (twice repeated) will reach from the Length of the Cask, to the whole Content of the Veſſel. This Gauge Point is at 17.2 for Ale, almoſt 19 for Wine.

Mr. *Mayne* (in his little Book, called his *Practical Gauger*, which I recommend as the plaineſt, and eaſieſt, and ſhorteſt I ever met with,) beſides very good Rules for Gauging all ſorts of Veſſels, and the uſe of the Gauging Rule, which is very well done. He hath (beſides theſe) 4 very uſeful Tables, which I ſhall here mention, but refer the Reader to the Book it ſelf.

1. The Firſt is a Table of Areas of Circles in Ale Gallons and Milleſimal Parts; to every Quarter of an Inch, from 10 to 144 Inches Diameter. This is to find the Contents of the Fruſtum of a Spheroid in Ale Gallons, *pag.* 29.

2. A Table

A Table of one Thirds of Area's of Circles in Wine Gallons, calculated to every Quarter of an Inch, from 10 to 60 Inches Diameter, *pag.* 39. This Table is to find readily the Contents of a Wine-Hogſhead, or Beer-Barrel in Wine Gallons.

3. A Table of the Contents of Cylinders in Ale Gallons, and Centeſimal Parts from 12 to 60 Inches Diameter, and to 8 Inches in Depth ready caſt up, ſo that you have nothing to do but to find the Plain Diameter, or the Mean Diameter of any Cylindrical Veſſel.

4. In the Fourth place, A Table of Area's of Segments of a Circle: The Radius divided into 100 parts, calculated to the $\frac{1}{1000}$ part of a Square Inch. Hereby you may find the Vacuity, Ullage, or Wants in a Cask partly full, lying with his Axis parallel to the Horizon: The Cask being taken as the Fruſtum of a Spheroid: The Table and uſe of it follows.

A TABLE of Area's of Segments.

V.	Area	V.	Area	V.	Area	V.	Area
1	.0017	26	2066	51	5127	76	8155
2	.0048	27	2178	52	5255	77	8263
3	.0087	28	2292	53	5382	78	8369
4	.0134	29	2407	54	5509	79	8473
5	.0187	30	2523	55	5636	80	8576
6	.0245	31	2640	56	5762	81	8677
7	.0308	32	2759	57	5888	82	8776
8	.0375	33	2878	58	6014	83	8873
9	.0446	34	2998	59	6140	84	8967
10	.0520	35	3119	60	6265	85	9059
11	.0598	36	3241	61	6389	86	9149
12	.0680	37	3364	62	6513	87	9236
13	.0764	38	3487	63	6636	88	9320
14	.0851	39	3611	64	6759	89	9402
15	.0941	40	3735	65	6881	90	9480
16	.1033	41	3860	66	7002	91	9554
17	.1127	42	3986	67	7122	92	9625
18	.1224	43	4112	68	7241	93	9692
19	.1323	44	4238	69	7360	94	9755
20	.1424	45	4364	70	7477	95	9813
21	1527	46	4491	71	7593	96	9866
22	1631	47	4618	72	7708	97	9913
23	1737	48	4745	73	7822	98	9952
24	1845	49	4873	74	7934	99	9983
25	1955	50	5000	75	8045	100	

The

The Uſe of this Table is as follows.

1. It is Requiſite that the Bung and Head Diameters, the Cask Length, the whole Content, and the Dry and Wet Inches be all known, and then if the Queſtion be

What is wanting, or what is Remaining in the Cask?

Divide accordingly either the Dry or Wet Inches by the Bung Diameter, and the Quotient ſeek in the Table under V, or Verſed Sine; againſt it ſtands a Number, which multiplyed by the whole Content exhibits the vacuity, if your Dividend were the dry Inches; or ſhews the Remaining Liquor, if your Dividend were the wet Inches.

Suppoſe the Bung Diameter 28, the Content of the Cask 60 Gallons, and Dry Inches 7.

Divide 7 by 28, by adding two Cyphers,

7ØØ(25
288
2

ſeek this 25 in the Table, over againſt it you find 1955, which Number multiply by the whole Content 60, ſo is the Ullage or Wants, 11.7300; cutting off the laſt

 4 Figures

4 Figures is 11 Gallons, and almost $\frac{3}{4}$ of a Gallon.

Now if the Question be, What Quantity of Liquor is remaining in this Cask?

Divide the Wet Inches by the Bung Diameter, that is 21 the Wet Inches by 28 thus, 28 | 2100(75. Now against 75 in the Table you find 8045, which being multiplyed by 60 the whole Content of the Cask.

The Remaining Liquor is	48.2700
The Wants was	11.7300
The whole Contents.	60.0000

Now if after Division there happen a Remainder or Fraction, and that be above Half the Divisor, I take the next bigger Number: Or if it be less than half the Divisor, I take the same Number which is in the Quotient.

The Description and Use of the Gauging Rule.

This Rule is commonly 4 Foot long, and is made to double in 4 joints for convenient Portage: It hath also 4 sides, on which are drawn several Lines. 1. There

1. There are two Lines called Diagonal, the one for Ale, the other for Wine Meaſure; Put the end which is cut ſlope-ways in at the Bung-hole, and let it touch the bottom of the Head; and the Number at the Bung ſhews the Ale or Wine Gallons reſpectively: This will give a very good eſtimate of the Content of all Cask in the form of a *London* Beer-Barrel, or the *French* Wine Hogſhead.

2. On another ſide there is put a Line of Inches from 1 to 48, each Inch decimally divided: And alſo upon the ſame ſide you have *Oughtred*'s Gauge Line, it being a Line of one Thirds of Area's of Circles in Wine Gallons, by which you may Gauge a Cask after this manner,

Put your Rule down at the Bung perpendicularly, obſerving what Number appears juſt even with the inſide of the Cask, ſuppoſe it 7: ſet that down twice, then take the Diameter at the Head, and let that ſhew you 6 upon the ſame Line, ſet that down to the former, add theſe three Numbers together, and multiply the Sum by the Cask length ſuppoſe 30, then cut off one place from the Product towards the Right Hand, and the Figures towards the left hand are your Number of Wine Gallons contained in that Cask.

If your Diameter fall amongst the Divisions between the Numbers, you must cut off 2 places from the Product. *Example.*

Bung Diameter	71	7
	71	7
Head Diameter	58	6
	200	20
Cask Length.	30	30 Length.
Content	60.00	60.0 Content.

3. On a third Face of this Rule is a Line of equal Parts numbred from one to 96: This Line considered together with that last before-mentioned, (*viz. Oughtred's* Gauge-Line) do make a Table of Area's of Circles in Ale-Gallons; so that if you find your Diameter in this Line, turn up the other Face, and against your Diameter you shall have the Area of your Circle in Ale-Measure. As for *Example.*

The Diameter of a Circle is 19 Inches, the Area of that Circle upon the other edge in *Oughtred's* line is a little above one Gallon, so the Diameter 30 Inches, the Area is 2.5 Gallons, so the Diameter 67 inches, it holds 12.5 Gallons upon one Inch of Depth.

The use of these lines thus together is the

the ſame with that of the Table of Areas of Circle in Ale Gallons, *pag.* 124.

4. On the fourth ſide of this Rule is drawn a line of Numbers, vulgarly called *Gunters* Line, being a Line of Logarithmes, whoſe uſe I have deſcribed In Inſtrumental Arithmetick, *pag.* 37, 38, 39, 40: Only in this place take notice as to Gauging, that at 17.2 you have a ſmall Braſs Pin, whereon to ſet the foot of your Compaſſes, and is called the Gauge point for Wine Gallons marked *W. G.* And another ſmall Braſs Pin at almoſt 19 the Gauge point for Ale Gallons, marked *A.G.* by which they are eaſily known. Now to Gauge a Cask by this line, you muſt firſt find the Diameter at the Head and Bung, and alſo the Casks length by the line of Inches: Theſe being had, find your Mean Diameter, by adding double the Bung Diameter, to once the diameter at the Head, and divide their Sum by 3, the Quotient take for your Mean diameter: Then with your Compaſſes ſet one foot in the Gauge point, whether it be for Wine or Ale reſpectively, and Extend the other to the mean diameter upon your Line of Numbers, and keeping the points at that diſtance, ſet one foot at the Number, expreſſing the Casks Length, and from thence double the diſtance of the feet of Compaſſes, exhibits the Content in Ale

or Wine Gallons reſpectively : As for *Example*, Bung diameter 27, Head
27 diameter 24, Length 30 Inches,
27 the mean diameter will be 26,
24 which in Ale Gallons 57, in
— Wine Gallons 69, and having
78(26 the mean diameter of a Conical
33 Tun (28½) depth of Liquor (29)
The Quantity of Ale Gallons will be found (63½) after the ſame manner, and is an Approximation near enough the Truth for common practice.

5. There is alſo another Line that runs parallel with this Line of Numbers, and is called a Line of Segments, but I do not like the Hypotheſis upon which it is framed; the way by the Table of Areas of Segments being more perfect; However it being upon the Rule, I ſhall ſhew you how to find the Ullage or wants in a Cask by this Line and the Line of Numbers.

Suppoſe the Bung diameter 24 Inches, wet 18 inches, Content 50 Ale Gallons, What is the Ullage or Wants in this Cask?

As 24 on the Line of Numbers is to Radius on the Segments, ſo is (6) the dry inches on the Line of Numbers to 17. 8 on the Segments. Then as Unity to 50 on the Line of Numbers, ſo is 17.8 on the Numbers to 8. 9 Ale Gallons, the wants required. So much for Gauging.

CHAP.

CHAP. XIV.

NOW besides this kind of measuring a Triangle, by taking the whole Content both of a Superficial and Solid Triangle; There is another way of Measuring a Triangle, or of finding the length of some of its sides, or the Quantity of some of its Angles by other Sides and Angles given or known.

For every Triangle consisting of 3 sides and 3 Angles, any 3 of these 6 parts being known, the rest may be easily found, either first Geometrically by Protraction, *pag.* 85, 86, 87: Or Secondly, Arithmetically, by the Rule of Proportion, as it is performed Logarithmetically, *pag.* 31, and Instrumentally, *pag.* 41. This I call properly the Resolution of a Triangle, or Trigonometry.

Many Learned Authors have made this very particular the subject of whole Books, as *Petiscius*, *Norwood*, and others; But for my present Purpose, I shall content my self with these following necessary Rules and Directions, which well applyed, contain in short the whole Art of Trigonometry.

OF

OF TRIGONOMETRY.

SECT. 1.

First then, By Protraction, or by *Scale* and *Compass*.

PROBLEM 1.

Having any 2 Angles and a Side included or contained betwixt them, to find the other Two Sides, and the Third Angle.

AS suppose the Angles given in the following Triangle were the right Angle at the Base 90 degrees, and the Angle at the Hypothenuse 30 degrees, and the Base included betwixt these two Angles 400. To find the length of the Hypothenuse,

nuse, and the length of the Perpendicular, as also the third Angle at the Perpendicular.

First, From a line of equal Parts take in your Compasses the Length of the Base 400, and drawing a Line at pleasure, prick the said distance upon the said line: And at one end of the Base protract an Angle of 30 degrees DE: At the other end of the said Base protract another Angle of 90 degrees, as FG, and drawing the perpendicular infinitely, and also the Hypothenuse, where these two intersect, or cut one another, doth confine the length of both the sides measured upon the same Line of equal Parts with your Compasses; and also doth include the Third Angle sought betwixt them, *viz.* 60 degrees.

PROBL. 2.

Having Two Sides, and the Angle included betwixt them, to find the other Two Angles, and the Third Side.

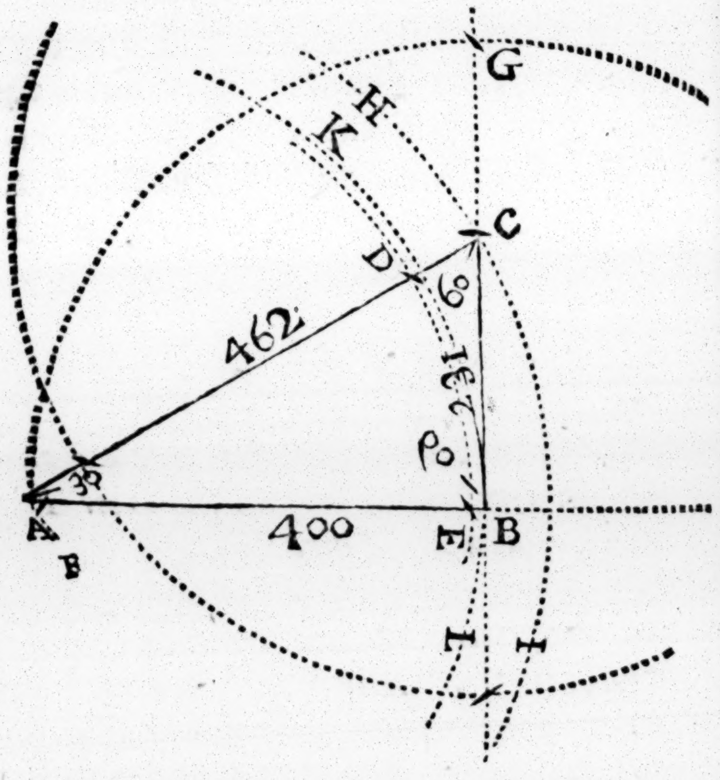
This you will find not to differ much from the former: Only first you must Protract the Angle, drawing the sides infinitely, and afterwards from the Angular point prick out either way their Exact Length, and from those Points draw the third side, which

which will include at either end the Two Angles sought.

PROBL. 2.

Having the Three sides given, to find the Three Angles of any Right Lined Triangle.

First, Take the distance of the longest side in your Compasses, and from any point as A, with that same distance draw an Arch of a Circle H I; then take the side



next the longest, and from the same point draw another Arch K L. Lastly, from any

any point in one of theſe Arches, to any other point in the other Arch, prick the length of the Third ſide, and drawing Lines from theſe three points from one to another, theſe Lines or Sides will include the three Angles required, A. B. C. A containing 30 degrees, B 90 degrees, and C 60 degrees.

This way of finding Sides and Angles by Geometry or Protraction, though it is not ſo exact as by Arithmetical Operation; yet it is ſufficient for ordinary Uſe, and of all other, both the moſt eaſie and demonſtrative, and alſo the moſt pleaſant and delightful.

Note, That in every right Angled Triangle, if you have one of the acute Angles given, the other is alſo given, becauſe it is the complement thereof to 90 degrees as the complement of 30 is 60. Alſo, if in any right Lin'd Triangle whatſoever you have any two of the Angles given, you have alſo the third Angle given, becauſe the three Angles of any right lin'd Triangle, is equal to a *S*emicircle, or 180 degrees.

SECT.

SECT. 2.

But in the Second Place, by some sides or Angles of a Triangle given, to find the Rest, (which is properly the Resolution of a Triangle) is the most exactly performed by Arithmetick.

PROBL. I.

First, By Common Arithmetick, Sides may be found by these Rules.

The Square of the Base and Perpendicular is equal to the Square of the Hypothenuse: As suppose the Base to be 4, the Square of 4 is 16; then suppose the perpendicular 3, the Square of 3 is 9; these 2 Numbers added together is 25: Now the Hypothenuse being 5, the Square of 5 is also 25. Likewise having the Hypothenuse, and one of the other sides, you may find the other side: As suppose the Hypothenuse 5, the Square of which is 25; as 5 multiplyed by 5 is 25: Then suppose the other side given be the Base 4, the Square of that multiplyed in it self is 16: This being deducted from the Square of the Hypothenuse 25, there Rests 9, which is the Square of the perpendicular 3: The same Rule to be obser-

observed if the Perpendicular had been given, and the Base required.

SECT. 3.

Having shewed you how to find Sides by *Common Arithmetick*, I shall proceed to shew you, How both Sides and Angles may be found by *Logarithmetical* and *Instrumental Arithmetick* by these following Rules. The first is more particular for Right Angled Triangles only, the other two more general for all plain Triangles whatsoever.

RULE 1.

1. *First then, having Two Sides, and a Right Angle betwixt them given, To find either of the Two Angles, and the Third side. The Proportion is,*

As the greater side given is to the lesser side, so is the Tangent of 45 degrees to the Tangent of the lesser Angle, and its complement to 90 is the greater Angle: As for Example, in the former Triangle, pag. 134: The Base being given 400, and the perpendicular 231, and the Right Angle 90 contained betwixt them: Then say by Logarithmes, as the Logarithme of 400 is to the Logarithme of 231, so is the Tangent

Tangent of 45 to the Tangent of the lesser Angle, (*viz.* 30 degrees) which deducted from 90, leaves 60 degrees for the other Angle: This is done by working by the *Rule of Three* in *Logarithmes*, as is taught *page* 31, but I shall give you another Example in this place.

If 400 give 231, what gives Tangent 45?

The Logarithme of 400 is	2.602059
The Logarithme of 231 is	2.363612
The Tangent of 45 is	10.000000
These 2 added together is	12.363612
From whence deduct the first	2.602059
Rests the Tangent of 30 deg.	9.761553

for the lesser Angle: The complement whereof to 90 degrees, is 60 the greater Angle.

The same may be done by Instrumental Arithmetick, or the Lines of Numbers and Tangents by the Rule of Three upon those Lines as is taught *pag.* 41; but I shall here add this Example more, Extend the Compasses from 400, to 231 in the Line of Numbers, the same Extent upon the Line of Tangents

Tangents will reach from 45 to 30 degrees for the leſſer Angle: the greater Angle being its complement to 90 *degrees, viz.* 60: And the two Angles found, and the two ſides given, you may eaſily find the third ſide by the Rule in *Common Arithmetick, pag.* 136, or by the third Rule following, to which I refer you.

RULE 2.

The ſecond Rule for reſolving of a Triangle is more general than the former, becauſe it reſolves not only a right angled Triangle, but alſo an Acute, and an Obtuſe Angled Triangle, that is any Right Lin'd Triangle whatſoever, and it is this.

As the Sum of two of the ſides is to the difference of the ſame ſides, ſo is the Tangent of half the Sum of the oppoſite Angles to the Tangent of half their difference.

Which I ſhall more fully explain by this Example, As ſuppoſe two ſides to be given, and any Angle betwixt them, to find either of the other Angles: Firſt, Add the ſides together, which ſuppoſe 303 and 176 makes 479: And Secondly, Subſtract the one from the other, their difference is 127. Now ſuppoſe the Angle given to be 110 *d.* 30 *m.*

Take

Take the complement thereof to 180, which is 69 *d.* 30 *m.* and this is the Sum of the 2 unknown Angles, (becauſe every right lin'd Triangle is equal to 2 right Angles, or 180 degrees.) Now the half of this 69*d.* 30 *m.* is 34*d.*45*m.* this being done, the Proportion is,

As the Logarithme of the whole Sum 479 is to the Logarithme of the difference 127, ſo is the Tangent of the half Sum of the two unknown Angles, *viz.* 34 *d.* 45 *m.* To the Tangent of half the difference between the ſaid Angles, *viz.* 10 *d.* 25 *m.*

For the ſaid 10 *d.* 25 *m.* being added to the half Sum (34 *d.* 45 *m.*) the Sum will be 45 *d.* 10 *m.* the quantity of the bigger Angle; and being ſubſtracted from the ſame, will ſhew the Quantity of the leſſer Angle, *viz.* 24 *d.* 20 *m.*

The Proportion may be wrought either by the *Rule of Three* in Logarithmetical; or by the Lines of Numbers and Tangents in *Inſtrumental Arithmetick*, *pag.* 31. and 41.

RULE 3.

The third General Rule, which is the laſt, I ſhall lay down, but not the leaſt, is the moſt general of all the reſt, and indeed may ſerve inſtead of all; and by me has been accounted even all in all; inſomuch as I once thought to have given no other, and but only for varieties ſake there is no other abſo-

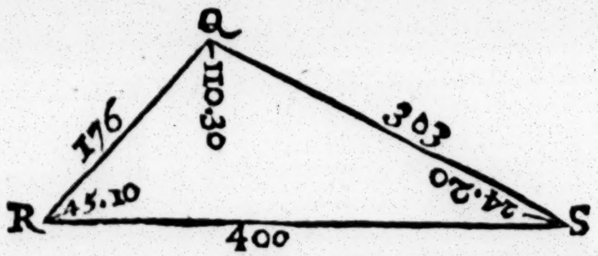
absolutely neceſſary, which for its excellency deſerves to be writ in Letters of Gold, which together with the Golden Rule or Rule of Three, as it is wrought and performed by Logarithmes, *pag.* 31, and by the lines of Numbers, Sines and Tangents, *pag.* 41, is ſufficiently able to do whatſoever is neceſſary in Trigonometry, which take as follows.

In all Triangles whatſoever, every ſide is proportional to its oppoſite Angle, and every Angle to its oppoſite ſide: And further, as the Angle oppoſite to one ſide is to the Angle oppoſite to the other ſide, ſo is the ſides themſelves to one another, *& è contra*, the ſides are to the Angles.

As for Example in the Triangle, *pag.* 134. As the Angle A 30 *d.* is to the perpendicular 231, ſo is the Angle C 60 *d.* to the Baſe 400, and ſo is the Radius or the right Angle 90 *d.* to the Hypothenuſe 462, *& contra*, ſo that there needs no more but having any Three of the ſix parts of a Triangle given, any one, or every one of the reſt may be found out by the Rule of Proportion, as it is performed by Logarithmes, *pag.* 31. or by the Lines of Numbers, Sines, and Tangents, *pag.* 41. though the Rule of Proportion in Common Arithmetick will not do it in Common Numbers, that is, will not find Angles. How to find Sides by Common

mon Arithmetick is taught before, *pag.* 136, But by Logarithmes or the Lines you may find not only Sides, but Angles allo: Taking this obſervation along with you, That when you meet with an Obtuſe Angle, to find its oppoſite ſide, you muſt work by its complement to 180, which being uſed inſtead of the Angle it ſelf, will effect the ſame thing in every reſpect. As to give an example in the Obtuſe Angled Triangle QRS, whoſe Angle at Q is obtuſe, being 110 *d.* 30 *m.* the Angle *R* 45 *d.* 10 *m.* the Angle *S* 24 *d.* 20 *m.* the ſide RS oppoſite to the obtuſe Angle is 400, the ſide QS. 303 oppoſite to the Angle R; The ſide QR 176 oppoſite to the Angle S, ſo the proportions ſtand thus,

As 24 *d.* 20. *m.* the Angle at S is to 176, ſo is 69 *d.* 30. *m.* (the complement of 110 *d.* 30 *m.* to 180) to 400: And ſo is 45 *d.* 10 *m.* to 303, *& è contra*: Therefore ſay by the Rule of proportion, either by Logarithmes, or by Lines:



If 24 *d.* 20 *m.* give 176, what gives 69 *d.* 30 *m.* or 45 *d.* 10 *m.* to find the ſides. Or,

If

If 176 give 24 *d.* 20 *m.* What gives 400, r 303 to find the Angles: see *pag.* 31 and 1, where you have the way of working hese, or any such like proportions by the *ule of Three*, either in Logarithmes, or y the Lines.

CHAP. XV.

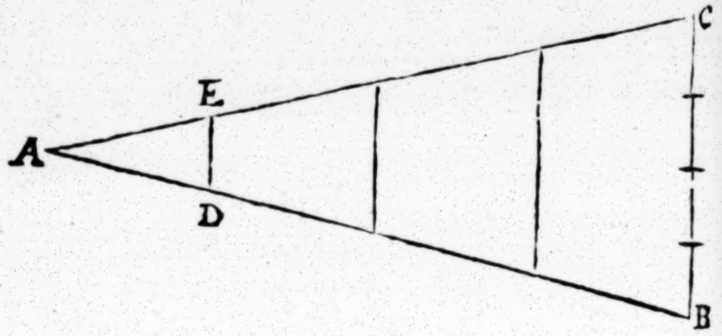
HAving now handled the three principal Geometrical Figures, the *Circle*, *quare*, and *Triangle*; how they are Produced, Reduced, and Measured themselves; nd consequently all other Figures or Bodies by their Rules; To conclude, What have to say in reference to Geometry in eneral, shall be to shew you the several xcellent uses of Mr. *Gunters* Sector, which eing properly Geometrical, I reserved for his place, to shut up general Geometry.

SECT. 1.

irst then, To shew the Ground of the Sector.

Let AB, AC represent the Legs of the ector: Then seeing these two, AB, AC re equal, and their Sections AD, AE also qual, they shall be cut proportionably, and if

if we draw the Lines BC, DE, they will be parallel by the Second *Prob.* 6 Book of *Euclid*, and ſo the Triangles ABC, ADE ſhall be equiangled, by reaſon of the Common Angle at A, and the equal Angles at the Baſe, and therefore ſhall have the ſides pro-



portional about thoſe equal angles, by the fourth *Prob lib.* 6. *Euclid.* Again the ſide AD ſhall be to the ſide AB as the Baſe DE unto the parallel Baſe BC,and by Converſion AB ſhall be unto AD, as BC unto DE. And by permutation AD ſhall be unto DE, as AB to BC, *&c.* ſo that if AD be the fourth part of the ſide, AB, then DE ſhall be the fourth part of his parallel Baſis BC,the like reaſon holdeth in all other Sections. And by the way, to underſtand the general aim and uſe of the *Sector*, it is to perform the ſame by Lines, which Arithmetick does by Numbers, as I ſhall ſhew you particularly: Now theſe lines which are found out by the Sector, are of two ſorts: Either firſt

Lateral

Lateral (or ſuch as are found upon the ſides of the Sector) as are the Lines, AB, AC: Or Secondly, Parallel ſuch as are the lines CB, DE from one ſide of the Sector to the other in its parallel points.

Having premiſed thus much in general, I ſhall ſhew you in particular the ſeveral uſes of the *Sector*.

Now you muſt underſtand that (beſides the Lines of Numbers, Sines, and Tangents upon the outward edge of the Sector, and a foot divided into 12 inches; as alſo another line of equal parts, or a foot divided into 100, or to be ſuppoſed 1000 parts upon the Inner edge of the Sector.) There are theſe ſeveral Lines upon the Sector it ſelf.

Theſe four, as alſo the 2 laſt are the principal Lines.	1. A Line of Sines. 2. A Line of Lines. 3. A Line of Superficies. 4. A Line of Solids.
Theſe 5 Lines are leſs principal, and for particular Uſes.	5. The Line of Quadrature. 6. The Line of Segments. 7. The Line of inſcribed Bodies. 8. The Line of Equated Bodies. 9. The Line of mettals.
The other two Principal Lines.	10. The Meridian Line. Laſtly, The Tangent Line.

And of theſe in the ſame order I here Rank or Reckon them, and firſt of the firſt (to wit) The Line of Sines.

SECT. 2.

The Line of Sines.

1. To Protract Angles by a Line of Right Sines is taught *pag.* 87, 88. And to avoid Tautology, I shall not here repeat it, but refer the Reader thereunto, being one excellent use of this Line.

2. If the Radius of a Circle be the same with the Lateral Radius or Semiradius of the Sector, the respective several Sines are the same as in this Line of Sines: But if it be either greater or lesser, set over the given Radius in the Line of Sines betwixt 90 and 90, and the Parallel Sines all along are the right Sines to the same Radius; and hereby you may divide any Line given, as a Line of Sines; And by the same Rule any sine being given, you may find the Radius thereof, by making it a Parallel Sine in its respective Number, the Radius will be betwixt 90 and 90.

3. To find the Chord of any Ark, observe that the Right Sine of the same Ark is half the Chord required; as the Sine of 10 is the Chord of 20, the Sine of 20 is the Chord of 40, *&c.* all along. The Radius or 60 degrees of a Line of Chords being set over as a Parallel betwixt 30 and 30 of

of the Line of Sines upon the Sector, mark this well.

4. To find the versed Sine of any Ark. If you reckon from 90 at the end of the Sector to 80 towards the Center, that distance is the versed Sine 10; from 90 of to 70 the versed sine of 20, and so of the rest. if the versed Sine required exceed a Quadrant, as suppose 130 degrees, First, Take the whole *Radius*, and as many degrees as it is above 90, (as in this Example it is 40) add thereunto from the end of the Sector towards the Center.

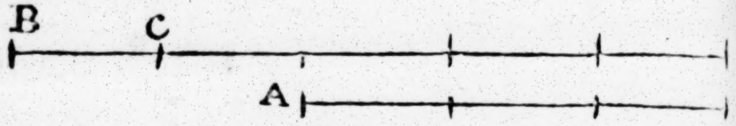
SECT. 3.

Of the Line of Lines.

This serves for a Line of equal parts upon any occasion, as they are numbred on the sides of the *Sector*, if the Radius be the same with the Lateral Radius. But if you would divide any other Line given into equal parts, set it over as a Parallel Radius betwixt 100 and 100, and the Parallel distances shall divide the same given Line into the same equal parts, according to the number which the points of your Compasses do stay upon: As if you would take or mark out 40 of those parts, the parallel distance betwixt 40 and 40 markt out upon the Line given, is 40 of the said parts, and so of the rest.

Or,

Or, Suppose the Line B were to be divided into 5 parts, First, I take the Line B betwixt the Compasses, and to it open the



Sector in the point 5 and 5, so the parallel between the point 1 and 1 doth give me the Line BC, which doth divide the same into the parts required.

PROBL. I.

To increase or Diminish a Line in a given Proportion.

As let A be a line given to be increased in the proportion, as 3 to 5. First, I take the line A in the Compasses, and open the Sector till the points of the Compasses do reach just between 3 and 3 on the sides of the Sector; so the Parallel between the points 5 and 5 doth give me the Line B, which was required. In like manner, if B were to be diminished as 5 to 3, First, set over B betwixt 5 and 5, and its parallel betwixt 3 and 3 gives you A diminished as 5 to 3.

PROBL.

PROBL. 2.

To find a proportion betwixt two or more right Lines given.

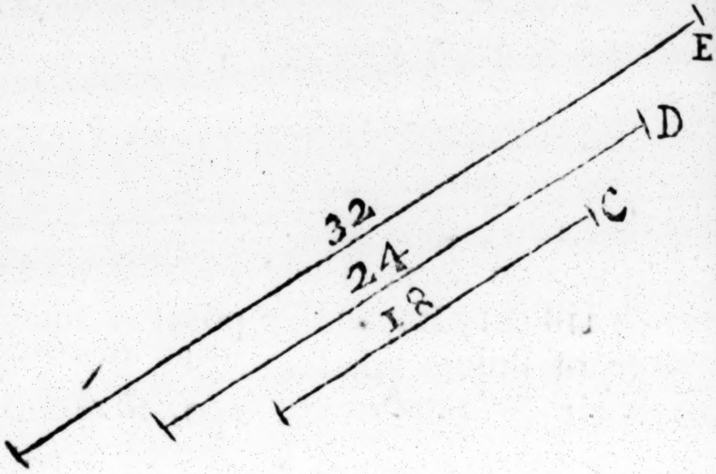
Take the greater Line given, and according to it open the *Sector* in the Points of 100 and 100; then take the lesser Lines severally, and carry them parallel to the greater till they stay in like points; so the number of Points wherein they stay shall shew their Proportion to 100, so the line B is in Proportion to A, as 100 to 60, and A to B, as 60 to 100.

PROBL. 3.

Two Lines given, to find a Third in continual Proportion.

Take the two Lines or Numbers given, and set them upon both sides of the Line of Lines (or equal Parts) from the Center, and Mark the Terms to which either of them extendeth; as suppose the Line C 18, and D 24 were the Lines or Numbers given. Measure with your Compasses the second Line D 24, and open the *Sector* at that distance in the Terms of the first Line or Number C, and so keep the *Sector* at this

Angle. The Parallel distance between the Terms of the second Lateral Number or



Line (24) being measured in the same Scale from whence his Parallel was taken, will give the third number proportional, *viz.* the Line E, containing 32 of the same equal parts.

PROBL. 4.

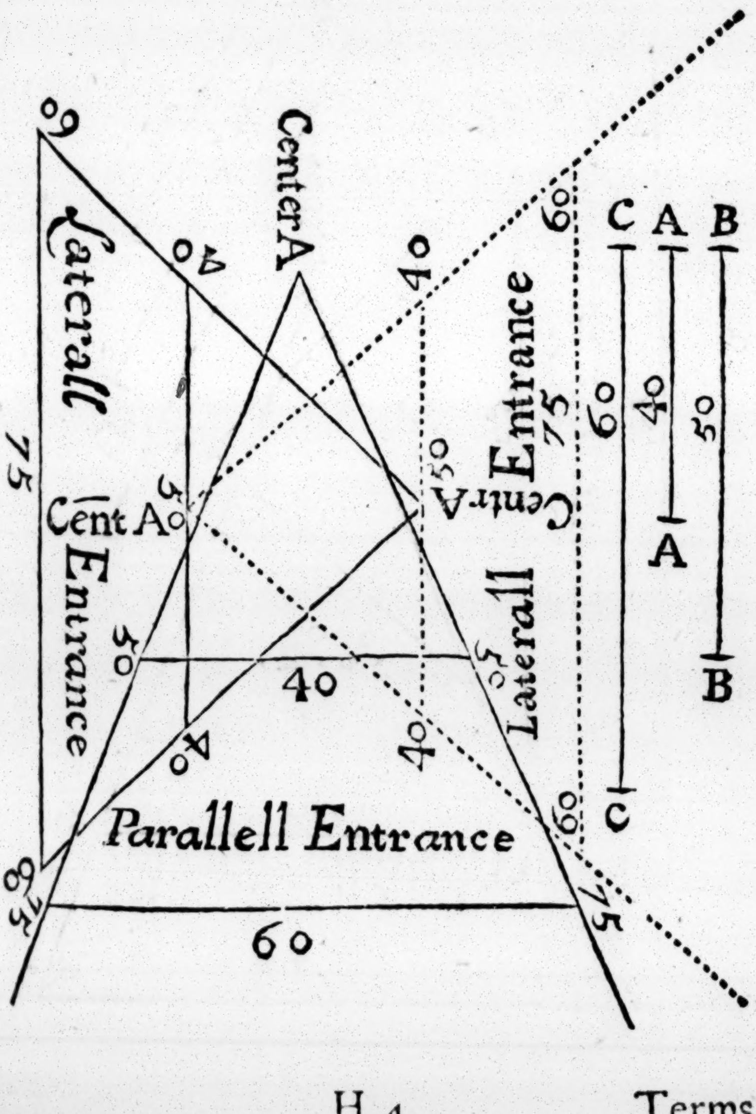
Three Lines or Numbers being given, to find a Fourth.

Let the Lines given be A 40, B 50, C 60, and the Question be this, If 40 Moneths give 50 *l.* What shall 60 Moneths give?

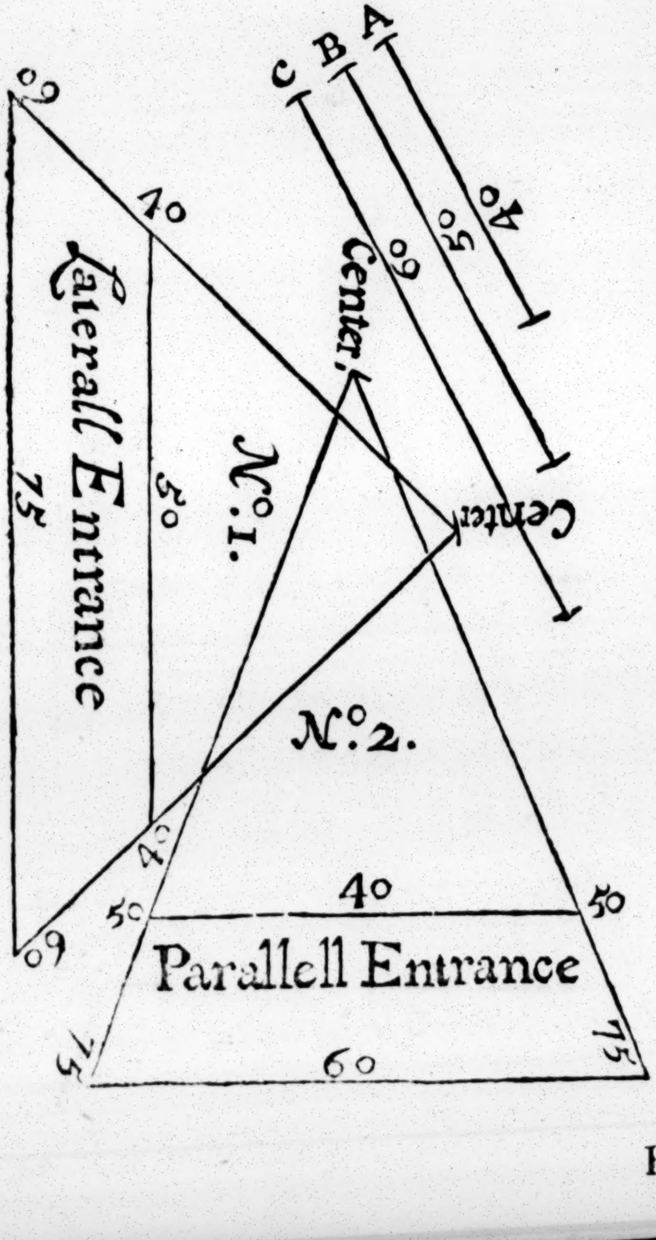
This you see is perfectly the Rule of 3, in

in the solution whereof a principal use of the *Sector* doth consist.

The first and third Lines or Numbers, (*viz.* A. 40. C 60.) must be placed on both sides the *Sector* from the *Center*, and with the second, *viz.* B 50. open the *Sector* in the



Terms of the first, *viz.* A 40. And the Parallel distance between the terms of the third, *viz.* C. 60, shall be the fourth proportional, to wit 75, as in the Example; and this is called Lateral entrance.



But

But if you would enter with a Parallel, do thus. First, with the first Line or Number A 40 in your Compasses open the *Sector* to that distance in the terms of the Second, *viz.* B. 50 (or make it the distance between 50 and 50 on the sides of the Sector.) Then take with your Compasses the third Line or Number C 60, and with that distance observe where the Points of the Compasses do stay in the Parallels, and that is the fourth proportional, which you will find 75, as before.

SECT. 4.

The Use of the Line of Lines, together with the Line of Sines, may be these.

PROBL. I.

In TRIGONOMETRY

Any Two Sides and an Angle included given, to find the Third Side.

Let the *Sector* be opened to the Angle given (30 *d.*) and observing the length of either side upon the sides of the Sector in the Line of Lines, the parallel distance from one point to the other, where the sides of the Triangle terminate, doth give the side

required. As ſuppoſe the Baſe of the Triangle *pag.* 134, be one ſide given, *viz.* 400, the Hypothenuſe the other ſide 462, the Angle included 30 at A, you will find the diſtance with the Compaſſes from 400, to 462 to be 231 the ſide required.

PROBL. 2.

Three ſides being given, to find an Angle.

Let the two containing ſides be laid on the Lines of the Sector from the Center one on one line, and the other on the other; and let the third ſide which is oppoſite to the Angle required be fitted over in the termes of the other two; ſo ſhall the *Sector* be opened in thoſe Lines to the Quantity of the Angle required. As for *Example* in the before mentioned *Triangle, pag.* 134.

Having

Having one of the containing sides 400 upon one side of the Sector, and 462 on the other, and setting these at the distance 231, the Angle at the Center will be found 30 *d.* Or suppose one side 72 in the Line of Lines, the other 54, and fit the opposite side (to the Angle required) over between 72 and 54; then with your Compasses measure the distance betwixt 50 and 50 in the same Line of Lines, and the same distance measured in the Line of Sines shall shew half the quantity of the Angle required.

As suppose the three sides given be AD 100, AC 75, AB 35. I might take 35 for the side CD out of the Line of Lines, and set it over from 100 to 75. This done, I take the distance between 50 and 50, and measuring it in the Line of Sines, I find it to be 8 *d.* 8 *m.* the double whereof is 16 *d.* 16 *m.* the Angle required.

SECT. 2.

The Use of the Line of Lines, together with the Line of Sines in Navigation, *see in its proper place,* That is,

1. To know how many Leagues do answer

ſwer to a Degree of Longitude in every ſeveral Latitude.

2. To know how many Leagues do anſwer to a Degree of Latitude in every ſeveral Rhumb. *Vide ut ſupra.*

SECT. 5.

The Uſe of the Line of Lines, together with the Line of Superficies in Extracting the Square Root.

1. The Line of Superficies is ſaid to be divided into 100 parts; therefore hath upon it twice the Figure 1, the former ſignifying Units, the latter Tens in that reſpect of the Line being divided into 100 parts.

2. Obſerve, If the Number, whoſe Square Root you deſire, conſiſt of even Figures, as 2, 4, 6, 8 Figures, or the like, the Number given ſhall fall out between the tenth diviſion and the end of the Sector, (or which is all one) between the latter Figure of 1, and the end of the Sector: But if the Number) whoſe Square Root you deſire) be odd, that is 1, 3, 5, 7 Figures, or the like, the Number given ſhall fall (or muſt be meaſured (between the Center of the *Sector*, and the *Tenth Diviſion*.

3. Theſe

3. Theſe being obſerved, ſet one foot of your Compaſſes in the Center, extend the other to the Term of the Number given in one of the Lines of Superficies, (which ſuppoſe 36.) The ſame diſtance applyed to one of the Line of Lines, ſhall find 6 for the Square Root.

Likewiſe to Square a Number given, firſt Extend the Compaſſes upon the Line of Lines to the Number you would Square (as ſuppoſe 6,) and apply the ſame diſtance to the Line of Superficies, will give (36) as before.

SECT. 6.

By the Line of Lines, and Solids, to find the Cube Root.

In finding a Cube Root of a Number given, prick the firſt Figure, and after that every third Figure from the right hand to the left; and if the laſt prick to the left hand fall under the laſt Figure, the Number given ſhall be reckoned at the beginning of the Line of Solids, from the Center to the firſt 1, and the firſt Figure of the Root ſhall be alwaies either 1 or 2.

If the laſt prick fall under the laſt Figure but One, Then the Number given ſhall be reckoned from the Center to the ſecond

ſecond 1, and the firſt Figure of the Root ſhall be alwaies either 2, 3, or 4.

But if the laſt prick fall under the laſt Figure but two, then the Number given ſhall be reckoned from the Center between the laſt Figure of one, and the end of the *Sector*: For you muſt know, as the Line of Superficies is ſuppoſed to be divided into 100 parts, and ſo having 2 Figures of 1 upon it, the firſt ſignifying Units, the ſecond Tens; ſo the Line of Solids is ſuppoſed to be divided into 1000 parts, and hath three Figures of one upon it, the firſt ſignifying Units, the ſecond Tens, and the Third Hundreds; ſo that according to this Diviſion the whole Line is 1000 as before. Theſe things being obſerved, the Rule for finding the Cube Root is this.

Set one Foot of your Compaſſes in the Center of the *Sector*; Extend the other in the Line of Solids to the point of the Number given according to the Rules before given: This diſtance applyed to the Line of Lines, ſhall ſhew the Cube Root of that Number.

Or to Cube a Number, Firſt, Extend the Compaſſes upon the Line of Lines; and then meaſure the ſame diſtance upon the Line of Solids, it will give you the Cube Number of a Cube Root given.

If the Reader desire further concerning these two last propositions of finding the Square and Cube Root by the Sector, I refer him to *Gunter* himself upon this Subject.

SECT. 7.

Of the Line of Superficies alone or single: As also the Line of Solids alone or single.

Observe first, That the Line of Superficies and Solids do serve to find Proportions betwixt Square and Cubique Bodies: As the Line of equal parts do betwixt lines (from thence called properly the Line of Lines) therefore the same Rules may in a manner serve for these which are given for the other right applyed, *mutatis mutandis.*

Secondly observe, That whatsoever is said concerning Superficies, the very same Rules serve for Solids, only that they are wrought upon different Lines. Therefore to avoid Repetition, observe to apply these Rules given for Superficies to Solids also, *mutatis mutandis* as I said before.

PROBL.

PROBL. I.

Therefore to proceed to find a Proportion between two or more like Superficies (as is before done in Lines,) do thus.

Take one of the ſides of the greater Superficies, and according to the ſaid ſide given open the *Sector* in the points of 100 and 100, then take the like ſide of the leſſer Superficies, and carry it parallel to the former, till they ſtay in like Points, ſo the Number of Points wherein they ſtay ſhall ſhew the proportion to 100 in the Lines of Superficies: As for Example, Let A and B be the ſides of like Superficies. Firſt, I take the ſide A in the Compaſſes, and to that diſtance open the Sector in the points of 100 and 100: Then keeping the Sector to that Angle, I enter the leſſer ſide B parallel to the former, and find it to croſs the Lines of Superficies in the point 40 and 40, therefore the Proportion of the Superficies, whoſe ſide is A, to that whoſe ſide is B, is as 100 to 40, or in leſſer Numbers, as 5 to 2. This Propoſition may alſo be wrought by 60, or any other Number that admits ſeveral Diviſions.

It may alſo be wrought without opening the Sector, for if the ſides of the Superficies

cies given be applyed to the Lines of Superficies, beginning alwaies at the Center of the Sector, There will be ſuch proportion found between them, as between the Number of parts whereon they fall.

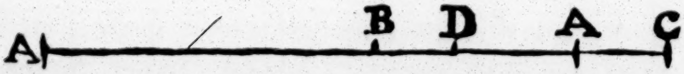
What is ſaid of Superficies, the like is to be underſtood of Solids wrought upon their own Lines.

PROBL. 2.

To Augment or Diminiſh a Superficies in a given proportion; or (which is the ſame) To add one Superficies to another, or ſubſtract one from another, will be all underſtood by this Example following.

Suppoſe A and B were the ſides of two Squares, or the Diameters of 2 Circles, and it were required to make a third Square or Circle equal to them both.

Firſt, The proportion between A and B will be found to be as 100 to 40, or in leſſer numbers, as 5 to 2; then becauſe 5 and 2 added together make 7, I augment the ſide A in proportion of 5 to 7, and produce the ſide C, on which if I make a



Square, or make it the Diameter of a Circle, the ſaid Square or Circle ſhall be equal to both the Squares of A and B, or to two Circles

Circles made upon their Diameters, which was the thing required. To substract one from another is after the same manner, as from 5 take 2, the Remainder is 3, so it must be diminished in proportion of 5 to 3, which is D.

PROBL. 3.

To find a Mean proportional between 2 Lines, which may be supposed the sides of a Square, or Diameters of a Circle, and thereby to find several Corollaries.

Suppose the Lines given be A A and AC. First find the proportion betwixt them as they are Lines as is taught *pag.* 60. which will be found to be as 4 to 9, wherefore I take the Line A C, and put it over to the Lines of Superficies between 9 and 9, and keeping the Sector at this Angle, his Parallel between 4 and 4, gives me A B for the mean Proportional: Or take the Line AA, and put it over in the Lines of Superficies between 4 and 4, and its Parallel between 9 and 9, will give AB for a Mean Proportional, which upon the same Scale of equal Parts will be found 6; for as 4 is to 6, so is 6 to 9.

Upon

Upon finding out this Mean Proportional depend many Corollaries, especially these following.

First to make a Square equal to any Superficies given.

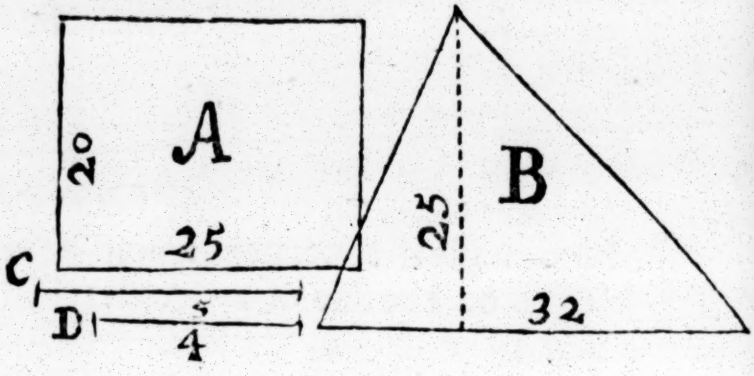
As a mean proportional between the unequal sides of a Parallelogram (or long Square:) or a mean proportional between half the Base, and the perpendicular of a Triangle shall be the side of a Geometrical Square equal to either of these in superficial Content. And if it be any other right-Lin'd Figure, it may be resolved into Triangles, and so a side of a Square found equal to every Triangle, and these reduced to one equal Square, it shall be equal to the whole Right-Lin'd Figure given.

Secondly, To find a Proportion between several Superficies though unlike to one another.

As for Example, The Oblong A, and the Triangle B are Superficies unlike to one another, therefore to find a Proportion between these.

First, Between the sides of A, *viz.* 20, and 25, I find a mean proportional, as C. This is the side of a Square equal to A, then between the perpendicular of the Triangle B, and half his Base, I find a mean proportional,

portional, *viz.* D. This is the ſide of a Square equal to B.



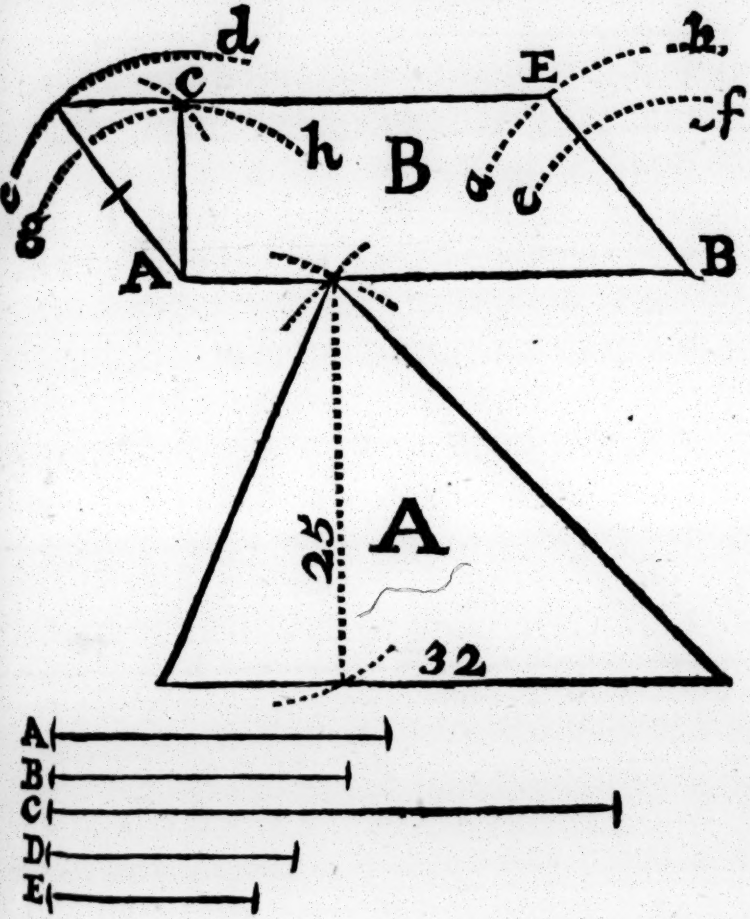
But the proportion between the Squares of C and D will by the Rule for it *pag.* 160 be found to be as 5 is to 4, *vide* the Rule it ſelf.

Thirdly to make a Superficies like to one Superficies, and equal to another.

Let one be the Triangle A, the other the Rhomboides B, Firſt, between the Perpendicular and the Baſe of B I find a mean proportional; and note it in B, as the ſide of a Square equal to B the Rhomboides. Then between the perpendicular and half the Baſe of the Triangle A find a mean proportional, and note it in A as the ſide of his equal Square.

Wherefore as the ſquared ſide of B is to the ſquared ſide of A, ſo ſhall the longer ſide

ſide of the Rhomboides given AB be to the longer ſide of the Rhomboides required, *viz.* C; and ſo ſhall the ſhorter ſide of the



Rhomboides given AD be to the ſhorter ſide of the Rhomboides required, *viz.* D. And ſo ſhall the perpendicular of the Rhomboides given AC be to the perpendicular of the Rhomboides required, *viz.* E. and having

ving the ſides and perpendicular, you may frame the Rhomboides up, and it will be equal to the Triangle A, which was the thing required.

And becauſe here is mention made of protracting or framing up a Rhomboides, having the longer ſide, ſhorter ſide, and perpendicular given, I ſhall here ſhew you how to do it.

Firſt then, Draw the length of the longer ſide, or the Line AB in the former Example. And then taking the Length of the ſhorter ſide in your Compaſſes, ſetting one foot in B, draw an Arch of a Circle *a* 6, as alſo ſetting one foot of your Compaſſes in A, draw the Arch *c d*. Then take the length of the perpendicular in your Compaſſes, and ſetting one foot in B, again draw the Arch *e f*. and removing to *A*, draw the other Arch *g h*. Theſe 4 Arches being drawn, lay your Ruler ſo as juſt to touch the Arches *e f* and *gh*, and draw a parallel Line from the point D, where the Ruler croſſes the Arch *c d*. to the point E, where the Ruler will croſs the Arch *ab*; and laſtly from the point D draw the Line D A, and from the point E draw the Line E B, and let fall the perpendicular AC from the point A, ſo will you have a Rhomboides as was required.

SECT.

SECT. 8.

Of the Lines of Quadrature.

The Lines of Quadrature may be known upon the Sector by the Letter Q, and by their place between the Lines of Sines.

Q ſignifies the ſide of a Square, 5 the ſide of a Pentagon, with 5 equal ſides, 6 of an Hexagon, that is, a Figure with 6 equal ſides, and ſo 7 for the ſide of an Heptagon, 8 of an Octagon, 9 of a Nonagon, &c. S ſtands for the Semidiameter of a Circle, and 90 for a line equal to 90 degrees in the Circumference of the ſame Circle.

PROBL. 1.

The uſe of theſe Lines may be to make a Square equal to a Circle given, or to make a Circle equal to a Square given.

If the Circle be given, take his Semidiameter, and to it open the Sector in the points at S; ſo the parallel taken from between the points at Q, ſhall be the ſide of the Square required: Or if the Square be given, Take his ſide, and to it open the Sector in the points at Q; ſo the Parallel taken from between the points at S, ſhall be the Semidiameter of the Circle required.

PROBL.

PROBL. 2.

To Reduce a Circle given, or a Square into an equal Pentagon, or other like Sided, and like Angled Figures.

Take the ſide of the Figure given, and fit it over in his due points, ſo the parallel taken from between the points of the other Figures, ſhall be the ſides of thoſe Figures, which being made up with equal Angles, ſhall be all equal one to the other.

Other Superficial Figures not here ſet down may firſt be reduced into a Square, and then into a Circle, or other of theſe equal Figures as before.

PROBL. 3.

In the third place, To find a Right Line equal to the circumference of a Circle, or other part thereof.

Take the Semidiameter of the Circle given, and to it (or to the ſame diſtance) open the Sector in the Points at S, ſo the parallel taken from between the points at 90, in this Line, ſhall be the fourth part of the circumference, *&c.*

SECT.

SECT. 9

Of the Lines of Segments.

The Lines of Segments are here placed between the Lines of Sines and Superficies, and are numbred by 5, 6, 7, 8, 9, 10.

The uſe of them is to divide a Circle given into 2 Segments, according to a proportion given, or to find a proportion between a Circle and his Segments given: For the Numbers 5, 6, 7, *&c.* do repreſent the Diameter of a Circle ſo divided into 100 parts, as that a right line drawn through theſe parts perpendicular to the Diameter, ſhall cut the Circle into two Segments, of which the greater Segment ſhall have that proportion to the whole Circle, as the parts cut have to 100: For Example, Let the Sector be opened in the points of 100 to the Diameter of the Circle given, as ſuppoſe BL, or any other Diameter; ſo a Parallel taken from the points proportional to the greater Segment required, ſhall give the depth of that greater ſegment: As if the Diameter of a Circle were B L, carry the greater ſegment LO when the Sector is opened in 100 and it will ſtay in 75, which ſhews the pro-

portion is as 75 to 100, or that the ſegment to the Circle is 3 parts of 4. Hereby you may find the ſide of a Square equal to any known ſegment of a Circle: For as the proportion is of the Segment to a Circle, ſo is the part of a Square to the Square equal to the whole Circle, which is taught before.

SECT. 10.

Of the Lines of Inſcribed Bodies.

Theſe are placed between the Lines of Lines, and may be known by the Letters D,S,I,C,O,T, of which D ſignifies the ſide of a Dodecahedron, I of an Iſcoſahedron, C of a Cube, O of an Octahedron, and T of a Tetrahedron, all inſcribed into the ſame Sphere, whoſe ſemidiameter is here ſignified by the Letter S.

The Uſes are

PROBL. 1.

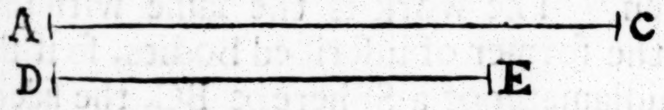
The Semidiameter of a Sphere being given, to find the ſides of the 5 Platonick or Regular Bodies, which may be inſcribed in the ſame Sphere.

PROBL.

PROBL. 2.

The side of any of the five Regular Bodies being given, To find the Semidiameter of a Sphere that will circumscribe the said Bodies.

If the Sphere be first given, take his Semidiameter, and to it open the Sector in the points at *S*, and the Parallel distance betwixt any other of the Bodies will give you the side of the same Body, which may be



inscribed in the Sphere: As if the Semidiameter be AC, the side of a Dodecahedron to be therein inscribed will be DE.

If the side of any other Body be first given, fit it over in his due points, and his parallel distance between the points *S* will give the semidiameter of a Sphere which shall circumscribe the same Body.

SECT. 11.
Of the Lines of Equated Bodies.

These are placed between the Lines of Lines and Lines of Solids, noted as the former with these Letters D.I.C.S.O.T, signifying the same things.

The Use is this,

The semidiameter of a Sphere given, to find the sides of the five Regular Platonick Bodies equal to that Sphere: Or the sides of any of the five Bodies given, to find the semidiameter of a Sphere, and the sides of the other Bodies equal to the first Body

given. The work is the same with that of the former of inscribed Bodies, so if the semidiameter of a Sphere be BC, the side of a Dodecahedron equal to this Sphere will be found FG, and of an Icosaedron H L. and so of all the Rest.

SECT. 12.

Of the Lines of Mettals.

The Lines of Mettals are here joyned with those of Æquated Bodies, and are noted with these Characters, ☉ ☿ ♄ ☽ ♀ ♂ ♃, of which ☉ *Sol* stands for God, ☿ *Mercury* for Quicksilver, ♄ *Saturn* for Lead, ☽ *Luna* for Silver, ♀ *Venus* for Copper, ♂ *Mars* for Iron, and ♃ *Jupiter* Tinn.

The

The uſe of them is to give a Proportion between theſe ſeveral Mettals in their magnitudes and weight, according to the Experiments of *Marinus Ghetaldus* in his Book, called *Promotus Archimedes.*

PROBL. I.

In like Bodies of ſeveral Mettals, and equal weight; having the Magnitude of the one, To find the Magnitude of the reſt.

Take the Magnitudes given out of the Lines of Solids, and to it open the *Sector* in the points belonging to the Metal given; ſo the Parallel taken from between the points of the other Metals, and meaſur'd in the Lines of Solids, ſhall give the Magnitudes of their Bodies.

PROBL. 2.

In like Bodies of ſeveral Metals and equal Magnitude, having the weight of the one, To find the weight of the reſt.

As if a Cube of Gold weighed 38 pounds, and it were required to know the weight of a Cube of Lead, having equal Magnitude: Firſt, I take 38 for the weight of the Golden Cube out of the Lines of Solids,

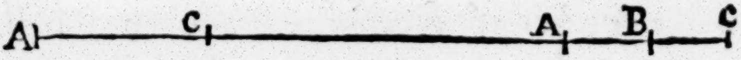
and put it over in the points of ♄ *Saturn* belonging to Lead; so the Parallel distance taken from between the points of ☉ *Sol* standing for Gold, and measured in the Line of Solids doth give the weight of the Leaden Cube required, to be 13½.

PROBL. 3.

A Body being given of one Metal, To make another like unto it of another Metal, and equal weight.

Let the Body given be a Sphere of Lead, containing in Magnitude 16 *d*, whose diameter is A A, to which I am to make a sphere of Iron of equal weight.

If I take out the Diameter AA, and put it over in the points of ♄ *Saturn* belonging to Lead, The Parallels taken from between the joints of ♂ *Mars* standing for



Iron shall be AB the Diameter of the Iron sphere required, and this compared with the other Diameter AA in the line of Solids, will be found to be 23 *d*. in Magnitude; C is the length of the whole line, and from A to C again.

PROBL.

P R O B L. 4.

A Body being given of one Metal, to make another like unto it of another Metal, according to a weight given.

As if the Body given were a ſphere of Lead, (or a Round Lead Bullet) whoſe Diameter is AA, and it were required to find the Diameter (of a ſphere of Iron) or of an Iron Bullet, which Iron ſphere ſhall weigh three times as much as the leaden ſphere or Bullet. I take AA and put it over in the points of ♄ which ſtands for Lead : His Parallel taken from between the points of ♂ ſignifying Iron, ſhall give me AB for the Diameter of an equal ſphere of Iron : If this be augmented in ſuch proportion as 1 to 3, it gives C for the Diameter required of a ſphere three times the weight of the former.

Having largely diſcourſed of *Geometry* in General, and of the three Famous Geometrical Figures, the *Circle*, *Square*, and *Triangle*, and having ſhewed you the Making and Meaſuring of theſe, and the Reducing of all other Figures, whether Regular, or Irregular, to ſome of theſe, if not to the two former, yet at leaſt to the laſt of theſe,

viz. a *Triangle*, and by consequence to Measure the same, by the same Rules that Measure a *Triangle* (or *Trigonometry*); Adding in conclusion the use of the *Sector*: I come now to Treat of *Geometry* according to the Proper and Genuine Etymology of the Word, which is derived from γῆ *Terra* & μέτρον *mensura*, that is, *The Measuring of the Earth*, and that (as elsewhere explained) to be understood the whole Globe of the Earth, containing *Sea* and *Land*: That which concerns the *Land* is called *Surveying*, and that which concerns the *Sea*, *Navigation*: And of these in particular. But first of Measuring this whole Globe of *Sea* and *Land* together in the General.

OF

OF THE GLOBE OF THE EARTH.

CHAP. I.

To Measure the whole Globe of the Earth comes under the Rules of the first Geometrical Figure, to wit, the Circle, and that a Solid Circle, (taking it for granted, That the Earth and Sea together make one Globular or Spherical Body.)

NOW we must find the solid Content of a Globe by the superficial Content of

a Globe, and the Superficial Content of a Globe by the superficial Content of a Circle, having the like Diameter. This Diameter, we must find by the circumference of the same Circle: And lastly, This Circumference (as all other Circumferences whatsoever) must contain 360 degrees. Now if we would take the circumferences of the Earth both Sea and Land in English Miles, we must know how many of these Miles upon the Earth are answerable to a Degree in the Heavens.

In the just Quantity and Measure of a Degree, the Common Rule is much out, which accounts 5 Foot to a Pace, and 1000 paces, *i. e.* 5000 feet to a Mile, and 60 such Miles, or 300000 Feet to a Degree.

The *English* Mile, which by the Statute is somewhat more, then this will not serve the turn, which is thus, $16\frac{1}{2}$ Feet make a Pole, 40 Poles a Furlong, 8 Furlongs a Mile, 60 such Miles contain 316800 Feet: By this account 1760 Yards is a Mile.

But Mr. *Norwood*'s Experiment by knowing the difference of Elevation of the Pole betwixt *York* and *London*, and measuring exactly the way betwixt them, passes for currant with all the Mathematicians I meet with of late, which was made on this manner. *June* 11. 1635, At *York*, by an Arch of a Sextan of more than 5 Foot semidiameter,

meter He took the Meridian Altitude of the Sun, which was 59 Degrees, 33 Minutes, Latitude 53 *d.* 58 *m.* *June* 11. 1633; in *London* he did the same, and found the Meridian Altitude of the *Sun* to be 62 Degrees, 1 Minute; in Latitude 51. 32: Now deducting the less latitude from the greater, there remains 2 Degrees, 28 Minutes, which is the difference of Latitude, (*York* being in 53 degrees 58 Minutes, and *London* in 51 Degrees 32 Minutes) as above noted: So by this was found 69 English or statute Miles; 4 Furlongs, 14 Poles, 9 Feet in a degree; or 367200 Feet. But to make 60 *English* Miles in a Degree, each Mile ought to contain 6120 Feet: These multiplyed by 60, make 367200 as above, and those again by 360 is 132192000 Feet for the circumference of the whole Earth.

Now according to this observation of Mr. *Norwoods*, there will be according to statute Measure 69 Miles, 4 Furlongs, 14 Poles and 9 Feet *English* in every Degree. Yet that we may not seem to differ so much from former Writers, and the Common Account we shall still reckon, only 60 *English* Miles to a Degree, and allow so many more feet to a Mile, *viz.* 6120 Feet, and 2040 yards; and if we cannot allow them the name of *English* Miles, we may call them Mathematical Miles, (though no question

question but our *Yorkshire* Miles may allow of the same Length.) Therefore,

First, 60 Miles allowed for a Degree, that multiplyed by 360, makes 21600 Miles for the whole circumference of the Earth.

Secondly, And by this Circumference to find the Diameter, Multiply 21600 by 7, is 151200, and that divided by 22, makes 6872$\frac{16}{22}$ for the whole Diameter (according to the Rule, *pag.* 78.) the semidiameter (rejecting the Fraction) is 3436 Miles to the Center of the Earth.

Thirdly, Now having the Diameter, to find the Area or superficial Content of a Circle having the like Diameter: (by the third Rule, *pag.* 78.) Multiply the Diameter by it self, that is, 6872 (omitting the Fraction) by 6872, it produceth 47194384 Miles; this Multiplyed again by 11, makes 519138224, and divided by 14, the Area of the Circle is 3708130 Miles.

Fourthly, This superficial Content of the Circle multiplyed by 4, (by the Rule, *pag.* 80.) produceth for the superficial Content of the Globe 148325204 Miles. And,

Fifthly, For the solid Content, Multiply the superficial Content by the sixth part of the Diameter, *viz.* 1145$\frac{1}{2}$, and it brings forth 169881600314^{2}, which are so many solid Miles; or the solid Content in Miles

(of

(of 6120 feet in a Mile) of the whole Terrestrial Globe, containing *Sea* and *Land*. And ſo much concerning the Meaſuring of the Earth in general: It follows that we Treat of *Surveying* and *Navigation* in particular? Only by the way take theſe two Corollaries.

1. Suppoſing the aforeſaid Diameter of the Earth $6872\frac{16}{22}$ Miles. If you imagine a hole through the Earth, and that a Milſtone ſhould be let fall down into this hole, and to move a Mile in a Minute of time, it would be more than two days and a half before it would come to the Center, and being there, would go no further, but as it were, hang in the Air.

2. Suppoſing the Circumference 21600 Miles, If a Man ſhould go every day 20 Miles, it would be three years wanting but a Fortnight before he could go once about the Earth.

And if a Bird ſhould fly round about it in two days, then muſt the motion be 450 Miles in an hour.

OF

OF SURVEYING

CHAP. II.

Surveying in a strict and limited sense may be taken only for the Measuring of some piece or parcel of Land: But in a larger sense, I take it here for the taking of all manner of Heights and Distances at Land, whether the same be Accessible, or Inaccessible.

TAking of Heights being a Resolving of a Triangle, standing upwards or vertically upon his Horizontal Base, and taken by a Quadrant with Thread and Plummet: Taking of Distances is a resolving of a Triangle (supposed to lye Horizontally Flat,) and taken by a Theodolite or such like Instrument, which is no other but a Quadrant layd Flat with

with an Index, and Sights inſtead of a Thread and Plummet: Not to exclude other Inſtruments, as the *Croſs-Staff*; for taking both Heights and Diſtances both at Sea and Land; but eſpecially at Sea: As alſo the *Plain Table*, an Inſtrument very much admired by many, of which you ſhall hear more hereafter.

As likewiſe I ſhall ſay ſomething of the Circumferentor, another Uſeful inſtrument for Surveying; but that which I do moſt admire and uſe, is the Quadrant for Heights, and the Theodolite for Diſtances, which is no more but a Circle, a Semicircle, or Quadrant layd Flat, divided into Degrees and Minutes with an Index and Sights, as any other Quadrant (properly ſo called) is held upright, and uſed with a Thread and Plummet.

Both theſe (that is) both the Quadrant and Theodolite may be notably ſupplyed by *Gunters* Sector, being ſet at the diſtance of a Quadrant (or 90 degrees) and having a looſe piece faſtned with two Mortices, and divided into Degrees for the quadrantal ſide, having alſo a Thread and Plummet faſtned in the Center, when you uſe it one way for a Quadrant, and a Pin for an Index and Sights to turn upon when you uſe it the other way for a Theodolite, being uſually each Leg a juſt foot in length

from

from the Center; the Quadrantal or looſe piece may be firſt divided into 90 degrees, and every degree into 6 parts, ſo that every one of the leſſer Diviſions are 10 Minutes, and theſe 10 may be ſuppoſed to be ſubdivided into other 10, which ſignifie ſingle Minutes, which is as much as may be for an Inſtrument of no larger Radius. When it is uſed as a Theodolite, there muſt be ſcrewed to it a ſocket of Braſs, whereby the whole Inſtrument may turn upon the top of a three leg'd ſtaff (hooped alſo with Braſs, as there ſhall be occaſion) with a ſcrew on the ſide of the ſocket to faſten the Inſtrument upon the ſtaff in any poſition you deſire: There may be alſo (to make it compleat) a Card and Needle ſcrewed upon the Center, and the Index which carries the ſights, being twice the Length of the Legs, (that is) 2 foot long, another of the ſame length without ſights may croſs it at Right Angles; ſo that making a perfect croſs, moving upon the Center as one part goes off the Quadrant, another comes on, which is as good as if it were a whole Circle. This may ſuffice for the deſcription of the Quadrant and Theodolite, I ſhall now ſhew you how to take Heights and Angles of Altitude by the Quadrant, and to take Diſtances or Angles of Poſition by the Theodolite: And to take both Heights and

and Distances by the Cross-staff, whether the same be Accessible, or Inaccessible.

SECT. 1.

To take any Height or Altitude, whose Foot or Basis is accessible at one station.

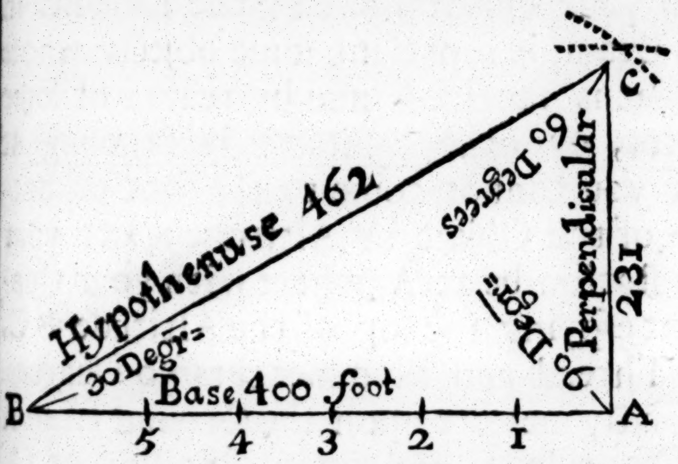
This is no more but the resolving of a plain Right Angled Triangle. As suppose the perpendicular CA were a Tree, Tower, or Steeple, or any other thing whose Height is required. First, standing at any convenient distance from the foot of the Object to be measured as at B, and there looking through both the sights of your Quadrant till you espie the top of the Altitude at C, observe what Degrees and Minutes are cut on the Limb of the Quadrant by the Thread, which suppose 30 degrees, as in the Example: These 30 degrees are the quantity of the Angle CBA, (or more plainly to call it the Angle B.) Now because that every right lin'd Triangle contains 180 degrees of a Circle, and every right Angle contains 90 of them; the Angle A being a right Angle, the other two Angles, that is B and C must contain other 90, whereof B by the Quadrant appears to be 30: Therefore C must needs be 60 degrees, the complement of 30 to 90. And lastly, You

You must measure by a Yard, Chain, or Two foot Rule, the Base of the Triangle, that is, the space or distance betwixt B, the place of your standing, and A the foot or bottom of the Altitude (which suppose to be 400 foot) As in the Example.

Now these three parts of the Triangle being given or known, that is, two Angles and a side, *viz.* the Angle B 30 degrees, and by consequence the Angle C 60 degrees, and the length of the Base 400 Foot: You may by Trigonometry and the Rule of Proportion find any of the other three parts of the same Triangle, *viz.* the Angle A, the Hypothenuse B C, or (that which is here required) the Height or Length of the Perpendicular CA. You will find that any Angle or side, (and consequently this side the perpendicular here required) may be found several ways, but by these three waies especially: First, By Protraction, *pag.* 85, 86, 87. Secondly, By Arithmetical Operation, *pag.* 137, 138. But especially by that general Rule in Trigonometry, *pag.* 141, that every Angle is proportional to its opposite side, and every side to its opposite Angle: Therefore as the Angle C 60 Degrees is to the known side BA 400; so is the Angle B 30 Degrees to its opposite side sought; the Perpendicular CA, or the Height required, which is no

more

more but performing the Rule of Three (or by theſe three Numbers given, to find a fourth ;) either firſt by Logarithmes, as is taught *pag* 41. Or, Secondly, By the Lines of Numbers, Sines and Tangents, *pag.* 41. Or laſtly, by the Sector, *pag.* 150, 151. by placing the Numbers on this manner.



If 60 degrees give 400 Foot, what gives 30 degrees? The Anſwer by all the forementioned ways will be found to be 231 *fere* the length of the perpendicular, or the Height required: But to avoid Repetition, I ſhall refer the Reader to the pages aforementioned for the manner of Operation.

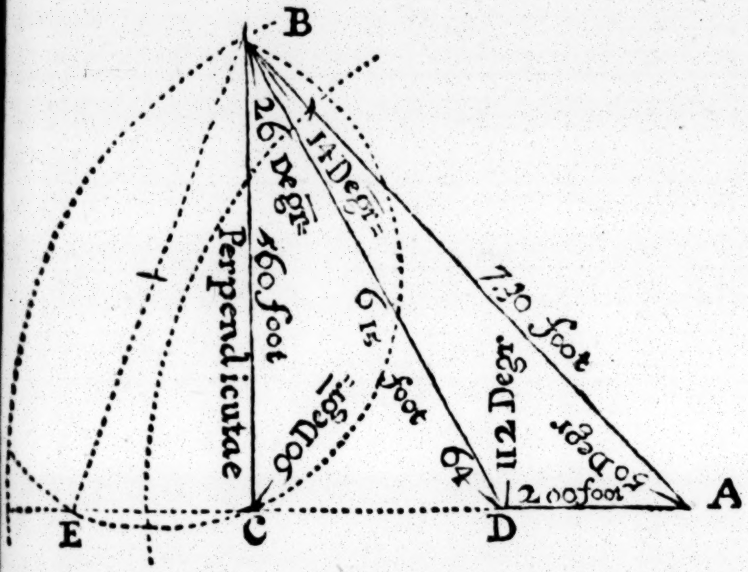
SECT.

SECT. 2.

To take an Inacceſſible Altitude at Two Stations.

For the effecting hereof, you muſt take two obſervations at two ſeveral Stations with your Quadrant. Let the Line BC in this Example repreſent ſome object, whoſe Height is required, and by reaſon of ſome Water, or other obſtacle interpoſing it ſelf, you cannot come to the foot or bottom of the Object or Altitude: Let your firſt ſtation be at A, where with your Quadrant eſpying the top of the Altitude B C, the Thread and Plummet cuts 50 Degrees. Secondly meaſure upon the Ground from A to D 200 foot, and again ſtanding at D, and eſpying the top of B C, obſerve what Degrees are cut by the Thread upon the Limb or Edge of your Quadrant, which ſuppoſe 64 *d.* Then fall to Protracting, and firſt draw an Angle at A 50 Degrees, and meaſuring 200 foot to D, protract there at D another Angle of 64 degrees, and from B, (that is from the point where the two lines AB and BD do interſect or cut one another,) let fall a perpendicular, as is taught *pag.* 55. *Probl.* 3 and meaſure the Length of the ſaid perpendicular by the ſame ſcale by

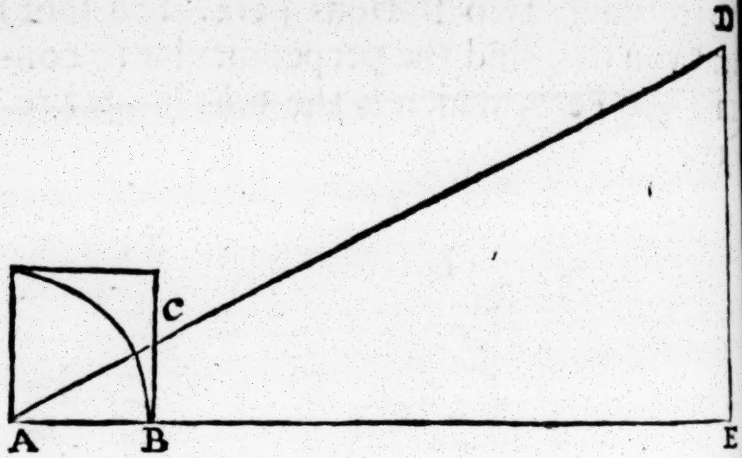
by which you laid down the diſtance betwixt your two ſtations (*viz.* 200 foot) and you will find the perpendicular to contain 560 Feet, which is the true Height required.



You may alſo do this by reſolving the two Triangles by Logarithmes, or by the lines of Numbers, Sines, and Tangents, &c.

Note that in taking all manner of Altitudes, whether acceſſible, or inacceſſible, you muſt alwaies add to the height found, the height of your Inſtrument from the ground.

Heights may be alſo taken by an Aſtrolabe: As alſo by a Quadrant, which repreſents the Tangents of a Circle (as in the following



following Example,) with an Index and Sights; for as the Radius A B is to the Tangent BC, so is AE the distance to ED the Altitude.

SECT. 3.

By the Height of the Sun, and length of the shadow, to find the Height of any Tree, House, Steeple, or the like.

First, Take the Suns Height by a Quadrant, which suppose 37 Degrees, and the length of the shadow of the said Tree, (which suppose 40 Feet) Then say as Radius to the length of the shadow of the Tree 40 Feet, so is the Tangent of the Suns height 37 Degrees to the height of the Tree desired, *viz.* $30\frac{14}{100}$ feet.

Note that when the Sun is 45 degrees high

high, then the ſhadow is equal to the height; at 26 *d.* 56 *m.* the length of the ſhadow is double to the height; at 18 *d.* 3*m.* it is 3 times; at 14*d.* 4 *m.* it is 4 times the Height, and at 11 *d.* 31 *m.* the ſhadow is 5 times the Height of any Altitude. Or, you may know the Height of any thing by a walking-ſtaff, or a Yard, or a two foot Rule by the ſhadow, thus.

Set the ſtaff upright, and meaſure how often the ſhadow of the ſtaff is longer or ſhorter than the ſtaff, ſo in proportion is the ſhadow of the Tree to the height of the Tree it ſelf. The like may be done by a Maſons Square ſet upright.

SECT. 4.

To take the Height of any thing acceſſible by 2 ſtraws or ſticks laid croſs one another.

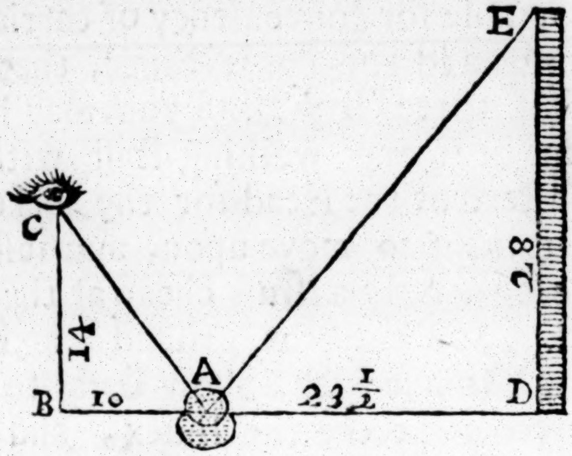
Take two ſtraws or ſticks of equal length, and faſten them perpendicularly croſs one another juſt in the middle of both, and go backwards or forwards till the bottom and top of the Altitude be even with the ends of the Croſs-ſtick, laying your Eye to the end of the other ſtick, and then is the height exactly as much as the diſtance meaſured to the Foot of the Altitude, adding to the Height found the Height of your Eye from the ground.

SECT.

SECT. 5.

To take an Altitude by a Bole of Water, or a ordinary Looking-glaſs.

Place on the ground a Bole of Water which done, erect your body ſtrait up, and go back in a right line from the building or other Altitude, till you eſpie in the Center or Middle of the Water the very top of it; which done, obſerve the place of your ſtanding, and meaſure the height of your Eye from the ground together with the diſtance of your ſtanding from the Water and the diſtance of the Water from the Baſe or Foot of the Altitude: Then ſay as the diſtance from your ſtanding to the water is to the Height of your Eye; ſo is the diſtance from the Water to the Foot of the Altitude, to the Height of the Altitude required. The ſame may be done by a Glaſs, if the Glaſs be placed Horizontal, or Flat upon the ground, and the Eye ſee the top of the Tower, *&c.* in the Angle or edge of the Glaſs, and in the Line of Reflection: I ſhall give you only one Example for both: Suppoſe the Bole of Water or the Glaſs be A which laid flat upon the Ground, or Horizontal; go backwards till you eſpie the top of the Altitude E, then meaſure how many



many parts is from your ſtanding at B to A, and alſo from B to your Eye at C, likewiſe the diſtance from A to D, and then ſay as the diſtance B A 10 is to the height of your Eye from the Ground CB 14, ſo is the diſtance from A to D the foot of the Altitude 23½ to the Altitude required, which you will find 28 of the ſame parts.

CHAP. III.

THus much may ſuffice for taking of Heights, in the next place we come to ſhew How to take *Diſtances*, and that (as I told you before) is done the beſt by the Theodolite, which is the moſt excellent

Instrument for that use (especially as I have contrived it upon the Sector) and also the most portable for conveniency of carriage; I shall only add this contrivance, that instead of a three leg'd staff you may have only an ordinary walking-staff with an Hoop instead of the Head for the socket of the Instrument to move upon, and instead of three legs, only a Brass cheap at the end to prick it down in the ground; and the staff may be made with hollow Cases to put in the cross peece, the Index, and the Sights, and if you please, a pair of Compasses, whereby it becomes a general Instrument both for observation and Calculation; the Sector you carry in your Pocket, and the Staff in your hand, appearing no other but an ordinary walking Staff, which for neatness and portability exceed any other contrivance I meet with.

SECT. 1.

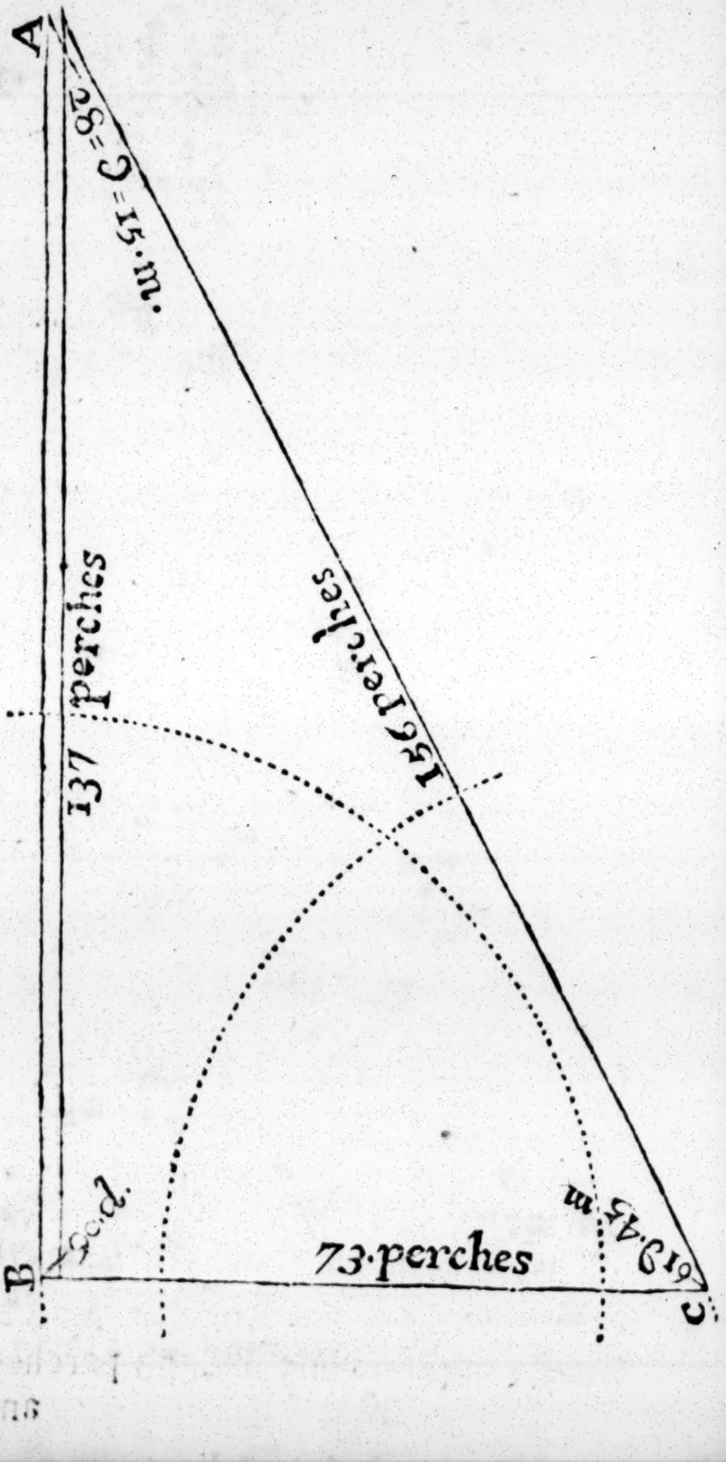
But to come to Practice, and by the Theodolite to find the Length or Distance of any place from you, how far soever it be.

This (as was formerly hinted) is no more but the resolving of a Triangle supposed to lye flat or Horizontally upon the Ground; As the taking of Heights is the resol-

resolving of a Triangle, standing vertically or upright upon his Horizontal Base: To shew you how to do it, take this Example.

Suppose you standing at B, and at A there were a Fort or Castle, or any other object, whose distance you desire from the place where you stand.

First, By your Theodolite placed at B lay the Index with the Sights upon the Diameter, where the Degrees take their beginning, and thorough the sights espie the Castle at A, the body of your Instrument so resting or screwed fast upon the staff, that it stands on, move the Index with the sights to 90 degrees, which make a Right Angle. There espie some other mark, as at C, then measuring the distance betwixt B and C, (which suppose 73 perches) remove your Instrument to C, your Instrument removed to C, there with the Index and Sights upon the Diameter where the Degrees take their beginning, Espy out your first station at B, then turning the Index with the sights till you again espie A, where note what Degrees are cut; suppose 61 *d.* 45 *m.* the complement whereof to 90 *d.* is 28 *d.* 15 *m.* for the Angle at A: Therefore you have two Angles and a side given to find another side of the Triangle, ABC; for you have the Angle at A 28 *d.* 15 *m.* and its opposite side BC. 73 perches,


A
28 d. 15 m.
137 perches
156 perches
90 d.
B
73 perches
61 d. 45 m
C

and again you have the Angle at C 61 *d.* 45 *m*, which holds the same proportion to its opposite side BA, which is the distance required.

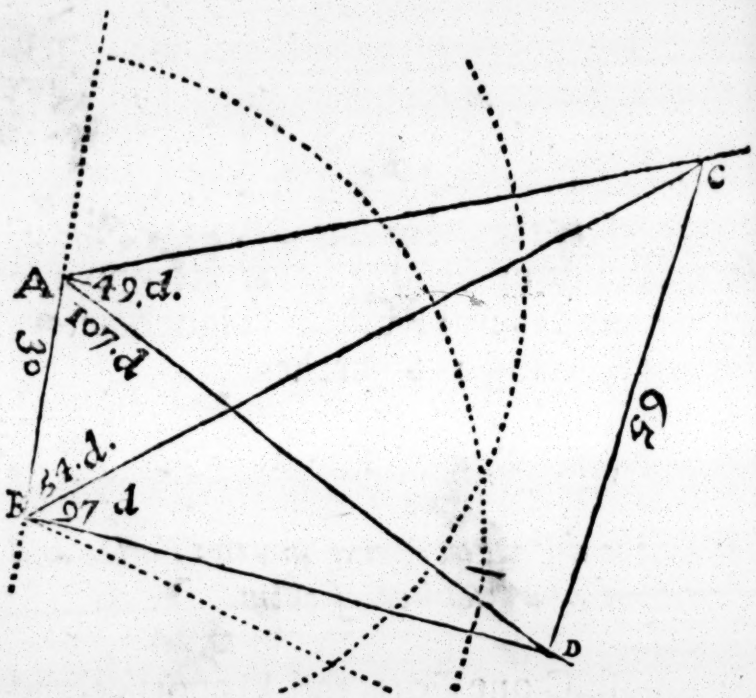
Which side or distance you may find by Resolving the Triangle either by Protraction, as in *pag.* 132, 133. or by Arithmetick working by Logarithmes, as *pag.* 31. or by the Lines of Numbers, Sines and Tangents, *pag.* 41. for as 28 *d.* 15 *m*, is to 73, so is 61 *d.* 45 *m.* to the side B A 137 perches: and (if the other side C A be required) so is Radius or 90 degrees to its opposite side by the general Rule in Trigonometry, *pag.* 141. being 156 perches.

SECT. 2.

To find the distance betwixt any two Forts, and yet come near neither of them.

Suppose C one Fort, and D another, and you are to find the distance betwixt them, and come nearer neither of them than A and B First place your Instrument at A, and laying your Index upon the Diameter where the Degrees take their beginning, through the sights espie the Fort at C; the Instrument so resting, move the Index till through the sights you espie D, which will be when the Index cuts upon the Limb 49

Degrees, which is the quantity of the Angle CAD; Then moving your Index till through the Sights you eſpie your ſecond ſtation at B, then you will find the Index to cut 107 Degrees, which is the quantity of the Angle CAB: Then meaſure the di-



ſtance betwixt A and B your firſt ſtation and ſecond, which ſuppoſe 30; then removing your Inſtrument to B, lay your Index with the ſights upon the Diameter where the Degrees take their beginning, and move the body of the Inſtrument upon the ſtaff, till through the ſights you eſpie the place of your firſt ſtation A, there ſcrew your

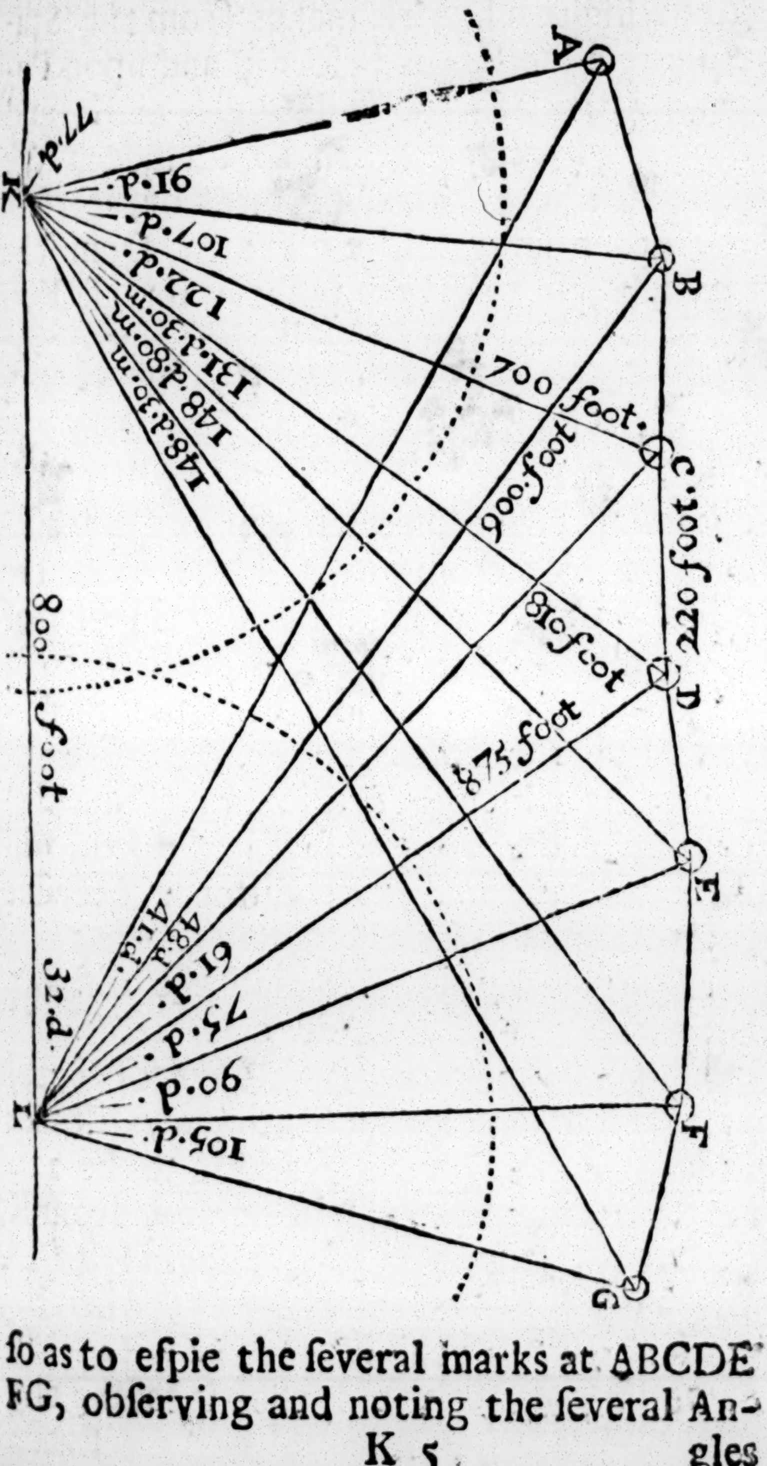
your Instrument fast, and move the Index till through the sights you espie the first Fort again at C, and observe how many Degrees are cut by the Index, which you will find 54 degres, or the Angle ABC: Lastly move your Index, till through the sights you espie the second Fort at D, and there observe what degrees are cut by the index, which you will find 97 degrees, or the Angle ABD. Note these several observations as you take them, and then upon Paper protract the same: As First, Draw your Stationary Line AB, and upon the point A protract the Angles 49 and 107 *d.* Likewise upon B protract the Angles 54 and 97 *d.* and lastly draw the line CD from the points of Intersection at C and D, and that measured upon the same Scale whence your Stationary distance was taken, will shew you the true distance betwixt the Forts C and D, which was the thing required.

SECT. 3.

How to take the Distance of divers places one from another according to their true Scituation in Plano, standing at some convenient distance.

Make choice of two convenient Stations (taken as large as you can, whereby your

work will be the truer,) from which ſtati-ons you may plainly diſcern all the marks or places whoſe Diſtances you would take: As ſuppoſe K and L to be your two Stati-ons; Firſt then, place your Inſtrument at L, and turn the whole Inſtrument about, till through the ſights on the Index lying upon the Diameter, (that is, where the Degrees take beginning) you eſpie your ſecond ſtation at K, which found, faſten your Inſtrument: Then moving your In-dex with the ſights, till you eſpie the mark at A, noting how many degrees are cut by the Index to the end you may protract the ſame when you have done your obſervati-on, and after the ſame manner move the Index, till through the ſights you eſpie all the reſt of the marks one after another, as BCDEF, noting down the ſeveral Angles from the Stationary Line KL. Then re-moving your Inſtrument from L to K, mea-ſure the Stationary diſtance betwixt L and K, which ſuppoſe 800 foot, as in the Ex-ample, when you come to protract, prick out this diſtance from a Line of equal parts upon the line K L, and placing your In-ſtrument at K, lay the Index upon the Diameter, and turn the whole Inſtrument about till through the back ſights you eſpie your firſt ſtation L, and there ſcrew it faſt; then moving the Index, direct your ſight



ſo as to eſpie the ſeveral marks at ABCDE FG, obſerving and noting the ſeveral An-

 gles

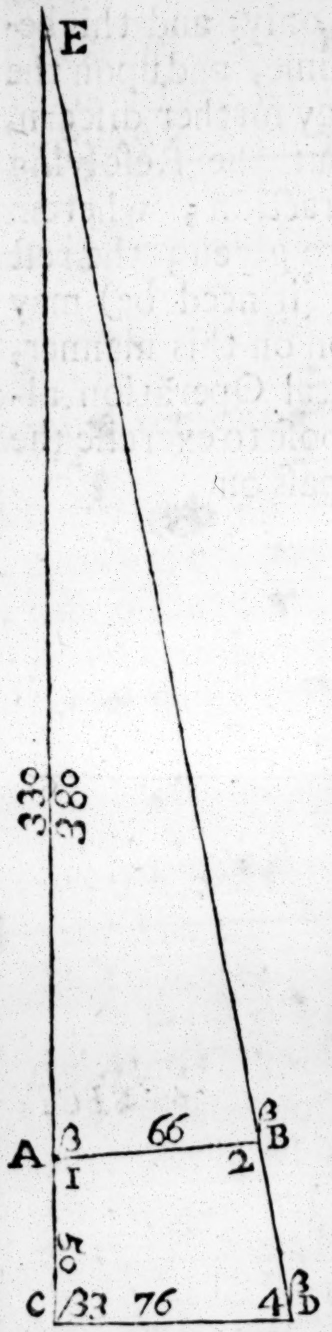
gles which each mark makes from the Stationary Line KL as before; and upon Paper protract all the ſaid Angles at K, and drawing Lines at length to include thoſe Angles, where theſe Lines meet or interſect the other ſeveral Lines drawn from the point L; in that very point of interſection make a mark, and ſet to it the name of the thing repreſented thereby; as if A be a Church-Steeple, B a Wind-mill, or the like: And after they be thus protracted upon Paper, meaſure with your Compaſſes the diſtance from each place from the point K or L, or the diſtance from one of theſe marks to another, as from A to B, from B to C, *&c.* Theſe diſtances meaſured upon the ſame Line or Scale of equal parts, by which the Stationary diſtance betwixt K and L were at firſt laid down, will ſhew the true and exact diſtance betwixt each place reſpectively.

As for Example, To find the diſtance betwixt C and D, meaſure it with your Compaſſes upon the ſame Line of equal parts, from which your Stationary diſtance was taken, and you will find it to be 220 foot, ſo from K to C will be 700, from K to D 810, from L to C 90, from L to D 875 foot, and ſo of all the reſt; whereby you may perceive this is the ſame with what is taught *pag.* 197; only that is taking the

diſtance

diſtance betwixt 2 places only, and this betwixt many at the ſame time, and upon the ſame ground; and you may further diſcern, that theſe are no more but the Reſolving of Triangles by Protraction, wherein two Angles and a Side are given; the reſt of the Sides and Angles (if need be) may be found as by protraction on this manner, ſo likewiſe by Arithmetical Operation alſo, which I leave on purpoſe to exercise the Readers Ingenuity, and paſs on.

SECT.



SECT. 4.

To take a long Distance without the help of any Instrument, unless it be only such an one that will direct you readily to set out a Right Angle, as any ordinary Carpenters Square, or the like.

Observe the Figure, and let C be your standing place, and let E be the mark afar off, whose distance from C you would know.

First, Move in a right Line between C and E to A any number of yards or perches, as suppose 50, and set up a Staff at A, then move in a perpendicular Line to C E from A to B, making a right Angle at A any distance, suppose 66, and set up another staff at B, then come back again to C, and remove

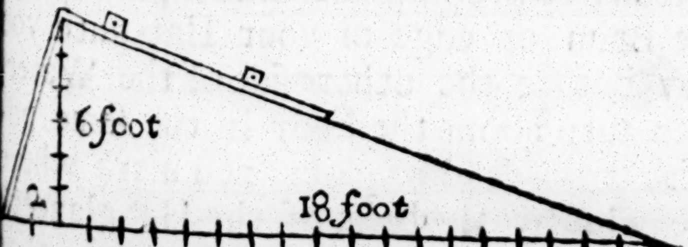
move in a perpendicular Line to CE, till you see the mark set up at B, and the point E in a Right Line, and set up another Staff at that place at D, getting the exact Distance thereof from C, which suppose 76; then substract the measured Distance AB 66 from the measured Distance CD 76, and note the Remainder, which is 10. Then by the Line of Numbers, or by the Rule of Proportion.

As the Difference between AB and BC 10 is to the distance between A and C 50, so is the measured Distance CD 76 to the Distance between C and E 380. Or, So is the measured Distance AB 66 to the Distance AE 330.

SECT. 5.

To take a Distance by a Carpenters, or Masons Square.

Take a Square with two Sights on one side, fasten it to the top of a Staff, which suppose 6 foot high, move the Square, so



that

that through the Sights you eſpie the thing which you deſire to know how far it is diſtant from you: Then obſerve the diſtance betwixt the Staff (faſtned to the other ſide of the Square where the ſights are not) and the 6 foot ſtaff which ſuppoſe 2 foot: Then obſerve as often as that diſtance is contained in the length of the ſtaff, ſo many times is the length of the ſtaff contained in the Diſtance required. As how many times 2 in 6, that is 3 times. Then ſay, 3 times 6 the length of the Staff is 18, the diſtance required. As in the Example.

SECT. 6.

To Meaſure an Inacceſſible Diſtance, as the Breadth of a River with the help of ones Hat only.

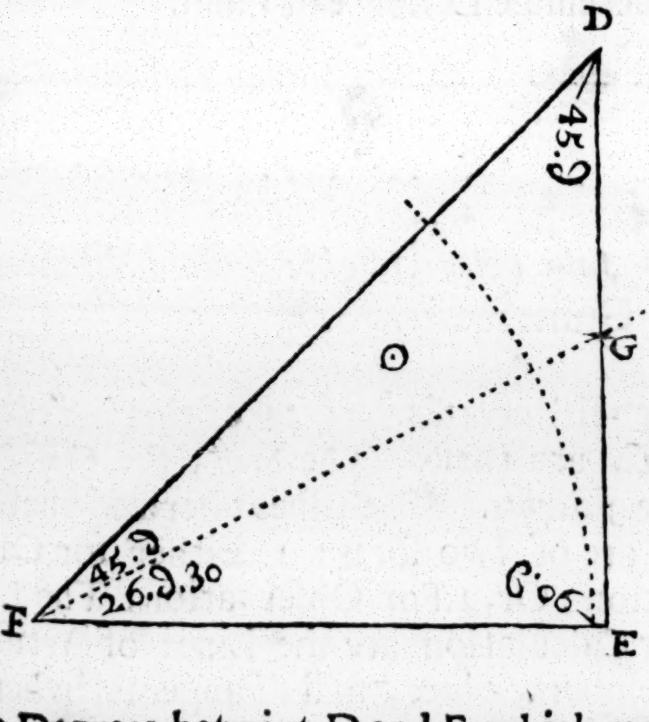
Having your Hat upon your Head, come near to the Bank of the River, and holding your head upright (which may be done by putting a ſmall ſtick to ſome one of your buttons to prop up the Chin) pluck down the Brim or edge of your Hat until you may but ſee the other ſide of the Water, then turn about the body in the ſame poſture towards ſome plain, and mark where the ſight by the brim of the Hat glanceth on the Ground: For the Diſtance from that place

place to your ſtanding is the Breadth of the River required.

SECT. 7.

How to take the Breadth of a River exactly.

Place your Theodolite, (or any other Inſtrument whereby you can take Angles;) at E obſerve ſome mark on the other ſide the River, as at D, and obſerve an Angle



of 90 Degrees betwixt D and F, which may be either on the Right Hand, or on the left, ſo far in the Line of 90 Degrees, till you

you eſpie the mark D doth juſtly make an Angle of 45 Degrees with the mark E, and this will be when you come to F; then meaſure carefully the Diſtance E F, and that ſhall be exactly equal to the Breadth of the River DE: An Angle of 26 *d.* 30 *m.* makes the Diſtance double to the Breadth, *&c.* and ſo by other Angles, but the beſt and ſureſt is 45 *d.* if you can uſe no other: So likewiſe by the ſame Rule may be taken the Diſtance of any Cape or Iſland from you at Sea, or any other Acceſſible Height, or Inacceſſible Diſtance at Land.

CHAP. IV.

To take both Heights and Diſtances by Gunters *Croſs-Staff.*

THE neceſſary parts of this Inſtrument are theſe, The *Staff*, the *Croſs*, and the 3 *Sights*. The Lines inſcribed thereupon are of two ſorts: 1. Either for Calculation: Or, 2. For Obſervation: The Lines for Calculation are the Lines of Artificial Numbers, Sines, and Tangents, whereof we have ſpoken largely, The Meridian Line for drawing of Sea-Charts; As alſo the Lines of verſed Sines, Chords, *&c.* which

which I shall say nothing of in this place. The second sort of Lines inscribed upon the Instrument are for Observation: First, A Line of Tangents for Observation of Angles. Secondly, A Line of equal parts not only for taking Angles, but also for taking perpendicular Heights and Distances. The Tangent Line on the Staff may be known by the double Numbers set on both sides of the Line, beginning at one side at 20, and ending at 90, and on the other side at 40, and ending at 180. On the Cross there are two Tangent Lines.

1. A Tangent Line of 36 *d.* 3 *m.* numbred by 5, 10, 15, to 35, the midst whereof is at 20 *d.* and therefore I call it the Tangent of 20, and this hath respect unto 20 *d.* in the Tangent on the Staff.

2. A Tangent Line of 49 *d.* 6 *m.* numbred by 5, 10, 15, to 45, the midst whereof is at 30 *d.* and hath Respect to 30 *d.* in the Tangent on the Staff, and therefore called the Tangent of 30. The Use of these follows.

SECT. I.

To find an Angle by the Tangent on the Staff.

Let the middle Sight be alwaies set to the

the middle of the Croſs, noted with 20 an 30, and then the Croſs drawn nearer th Eye, until the marks may be ſeen cloſe with in the ſights : For if the Eye at A (the en of the Staff) which is noted with 90 an 180, beholding the marks K and N betwee the two firſt ſights C and B, or the mark K and P between the two outward ſights the Croſs being drawn down to H, ſha ſtand at 30 and 60 in the Tangent of th Staff : it ſheweth the Angle KAN is 30 *d.* the Angle KAP 60 *d.* the one double t the other, which is the reaſon of the dou ble Numbers on this Line on the Staff; and this way will ſerve for any Angle from 20 towards 90 *d.* or from 40 to 180 *d.* but i the Angle be leſs than 20 *d.* we muſt then uſe the Tangents on the Croſs.

SECT. 2.

To find an Angle by the Tangent of 20 *on the Croſs.*

Set 20 unto 20, that is, the middle ſight to the midſt of the Croſs at the end of the Staff noted with 20, ſo the Eye at A behol- ding the marks L and N cloſe between the two firſt ſights C and B, ſhall ſee them in an Angle of 20 degrees : If the marks ſhall be nearer together as are M and N, then

then draw the Croſs in from C to E: If they be further aſunder, as are K and N, then draw out the Croſs from C to F, ſo the Quantity of the Angle ſhall be ſtill found in the Croſs in the Tangent of 20 *d.* at the end of the Staff; and this will ſerve for any Angle under 20, and ſo to 35 Degrees.

SECT. 3.

To find an Angle by the Tangent of 30 *upon the Croſs.*

This Tangent of 30 is here put the rather, that the end of the Staff reſting at the Eye, the hand may more eaſily remove the Croſs: For it ſuppoſeth the Radius to be no longer than AH, which is from the Eye at the end of the Staff to 30 *d.* about 22 inches and 7 parts, wherefore here ſet the middle ſight to 30 *d.* on the Staff, and then either draw the Croſs in or out until the marks be ſeen between the two firſt ſights: So the quantity of the Angle will be found in the Tangent of 30, which is here repreſented by the Line GH, and this will ſerve for any Angle from 0 *d.* towards 48 degrees.

SECT.

SECT. 4.

To take the Altitude of the Sun by Thread and Plummet.

Let the middle ſight be ſet to the midſt of the Croſs, and to that end of the Staff which is noted with 90 and 180; then having a Thread and Plummet at the beginning of the Croſs, and turning the Croſs upwards, and the Staff towards the Sun, the Thread will fall on the complement of the Altitude above the Horizon, and this may be applyed to other uſes alſo. Thus much for the Line of Tangents.

The other Line for Obſervation, whereby you may find both Angles and Sines is the Line of equal parts, which in *Gunters* upon the Staff is 36 inches, each inch divided into Tens, and each Ten into Halfs; and to anſwer it upon the Croſs, is 26 inches between the two outward ſights.

SECT. 5.

To take Angles by theſe Lines of equal parts.

If the Angles be obſerved between the two firſt ſights, there will be ſuch proportion between the parts of the Staff, and the

the parts of the Croſs, as between Radius, and the Tangent of the Angle; as if the parts intercepted on the Staff be 20, the parts of the Croſs 9: Then by Proportion, as 20 is to 9 upon the Line of Numbers, ſo is the Radius to the Tangent of 24 *d.* 14 *m.* upon the Line of Tangents: But if the Angle be obſerved between the two outward Sights, the parts being 20 and 9, as before, the Angle will be double to the former, *viz.* 48 *d.* 28 *m.*

SECT. 6.

To take not only Angles, but Sides alſo by this Line of Equal Parts, which is, To take Heights and Diſtances.

And Firſt, For Heights.

In taking of Heights, you are to hold the Croſs upwards, and for Diſtances, to hold the Croſs ſidewaies, and the Eye placed at the beginning of the Staff, eſpying the marks by the inward ſides of the Sights, there will be ſuch proportion betwixt the Diſtance and the Height as is between the parts intercepted on the Staff and Croſs: The Ground whereof you will find in *pag.* 189.

PROBL.

PROBL. I.

Having the Distance from your standing to the foot of the Altitude given, To find the Height.

As the Segment of the Staff is to the Segment on the Cross, so is the Distance to the Height. As if the Distance AB being measured, or known to be 256 feet, it were required to find the Height BC: First, I place the middle sight at 24 inches (or parts,) then holding the Staff level with the Distance, I raise the Cross parallel to the Height or Top of the Altitude; so as my Eye may see from A, (the beginning of inches, or parts upon the Staff) by the sight E unto the mark C; which being done, I find 18 inches intercepted on the Cross between the sights at E and D: Therefore I say, the Height BC is 192 feet; for as 24 is to 18, so is 256 to 192: Or if the observation were to be made before the Distance were measured, I would set the middle sight to 12, 16, 20, 24, or some such Number as might best be divided into several parts, and then work by Proportion: As in the former Example, Suppose I set the Staff to 24, and the Cross to 18, it shews the Height is $\frac{3}{4}$ of the distance, therefore the Distance being 256, the Height must be 192 as before.

PROBL.

PROBL. 2.

aving Part of the Height given, To find the whole Height.

As if the Height from G to C were own to be 48, and it were required to find e whole height BC. First, put the third ght, or some other sight upon the Cross tween the Eye and the mark G. Then, s the Difference between the sights E and 45, is to the whole Segment on the Cross D 180, so is the Part of the Height gi- en G C 48 to the whole Height BC 192.

PROBL. 3.

o find an Inaccessible Height at 2 Stations, by knowing only the Stationary Distance.

As the Difference of Segments on the taff is to the Stationary Distance, so is he Distance on the Cross to the Height equired: As suppose the first Station at H, the Segment on the Cross E D were 80, and the Segment of the Staff HD 00. Then coming 64 Feet nearer to B in direct Line towards the Height unto a econd station at A, and making another bservation: Suppose the Segment of the Cross

Croſs ED were ſtill 180 as before, and th Segment of the Staff AD 240. Take 24 out of 300, the Difference of Segments wi be 60: Then ſay,

As 60 the Difference of Segments t 64 the Stationary Diſtance (or Differenc of Stations) ſo is DE 180 the Segment o the Croſs to the Height required, *viz* BG 192.

PROBL. 4.

Having the Height of any thing given, to find the Diſtance.

Secondly, For finding Diſtances.

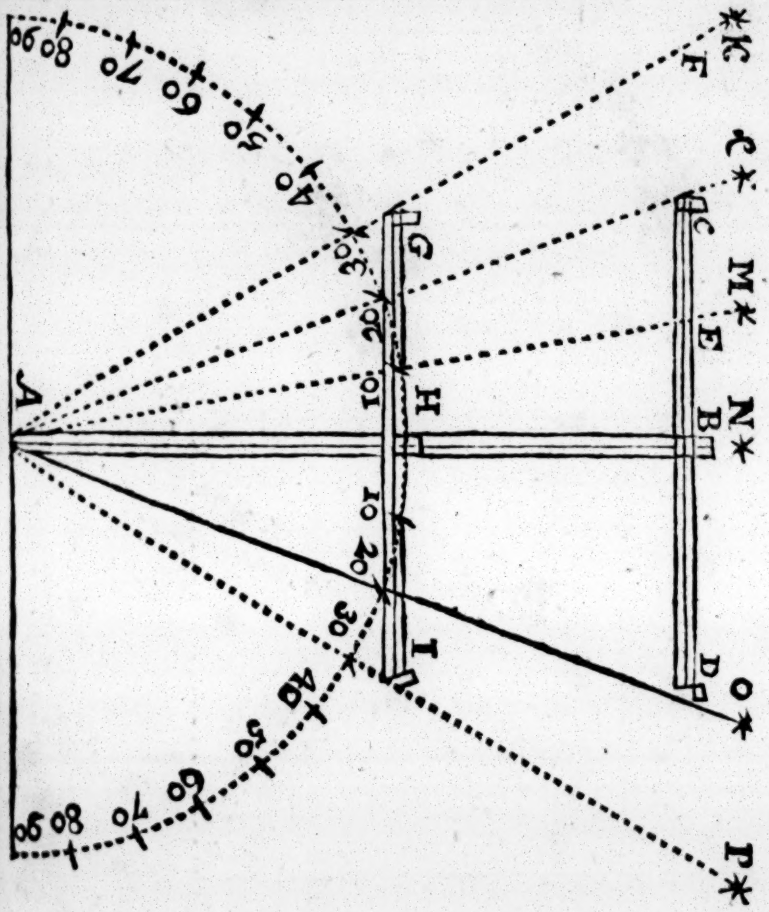
As the Segment of the Croſs (or the parts upon the Croſs which is all one) is to the Segment of the Staff, (or the parts upon the Staff,) ſo is the Height given to the Diſtance required. For Example, as the Segment ED 18 on the Croſs is to DA 24 on the Staff, ſo is CB 192 the Height to 256 the Diſtance.

PROBL. 5.

Having Part of the Height given, To find the Diſtance.

As the Difference between the Sights EF 45

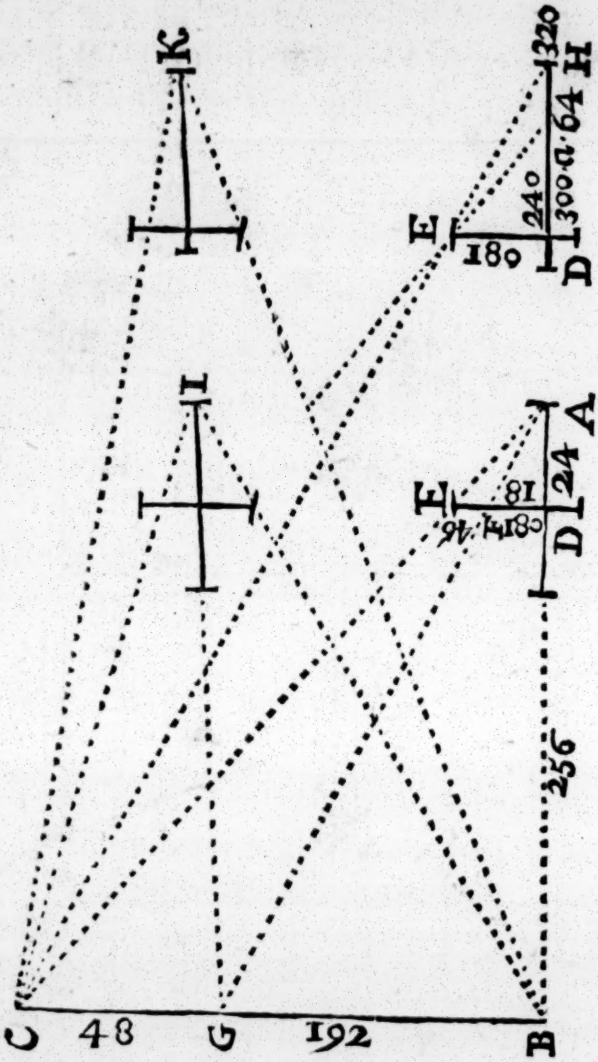
EF 45 is to the whole Segment of the Staff DA 24, ſo part of the Height GC 48 is to the diſtance required, *viz.* AB 256.



PROBL. 6.

To find a Diſtance at two Stations, by knowing only the Stationary Diſtance.

As the Difference of Segments on the Staff is to the Stationary Diſtance, ſo is



the whole Segment of the Staff to the distance required. The Segment of the Croſs being as before 180 The Segment of the Staff at the firſt Station 240, at the ſecond 300. The difference between theſe 60. The diſtance between the two Stations 64. Then ſay,

As

As 60 the difference of Segments is to 64 the Stationary diſtance, ſo is the Segment at the firſt ſtation 240 to the diſtance AB the firſt ſtation 256. And ſo is 300 the Segment at the ſecond Station, to 320, the diſtance HB the ſecond Station.

In taking a Breadth it is not material whether the two Stations be choſen at one end of the Breadth propoſed, or without it, or within it; If the Lines between the Stations be perpendicular unto the Breadth; as may appear if inſtead of the Stations at A and H , we make choice of the like Stations at I and K. To conclude, In all theſe there is regard to be had to the parallax of the Eye, and his Height above the Horizon; to the Semidiameter of the Sun, his Parallax and Refraction: As alſo the Parallax and Refraction of the Moon and Stars, if you will be ſo very exact; which you may ſee in this following Table.

The Table of theſe Parallaxes according to the Obſervations of *Ticho Brahe.*

The Table of Refractions of the Sun, Moon, and Stars.

Altitudes.	Sun Min.	Moon min.	Stars min.	Altitudes	Sun min.	Moon min.
0	34	33	30	18	6	6
1	26	25	21	19	5	6
2	20	20	15	20	4	5
3	17	17	12	21	4	5
4	15	15	11	22	3	4
5	14	14	10	23	3	4
6	13	13	09	24	3	4
7	12	13	8	25	2	3
8	11	12	7	26	2	3
9	10	11	6	27	2	3
10	10	11	5	28	2	2
11	9	10	5	29	2	2
12	9	09	4	30	1	2
13	8	09	4	31	1	2
14	8	8	3	32	1	1
15	7	8	3	33	1	1
16	7	7	2	34	1	1
17	6	7	2	35	1	1

The Refraction of the Sun, Moon, and Stars cauſeth them to appear higher above the Horizon than they really are, Therefore Subſtract the Refraction from the Altitude obſerved, that the true Altitude may be had.

The

The cause of these Refractions or Parallaxes is the Atmosphear, or the gross Vapours and Thickness of the Air near the Horizon, or the Face of the Earth; which to demonstrate by an easie experiment, set down an empty Bason on a Stool, laying a shilling in the bottom of it; and go so much backwards, till you bring the edge of the Bason and the Shilling in a right Line with your Eye, and then let another fill the Bason with fair Water, and you may go a pretty way further backwards, and still see the Shilling and the edge of the Bason in a right Line, the Refraction of the Water being the Cause thereof; and from this Reason we conclude that the Sun seems to appear above the Horizon when he is really set; and that other times he seems higher than he really is, as also the Moon and Stars.

CHAP. V.

HAVing shewed you several ways to take Heights and Distances in general, I come now to speak of *Surveying*, as it is taken in a strict and particular sense for the Measuring of Land according to the Diversity of Inclosures: Now this Measuring of

Land is no more but finding the Area or Content of a Superficies, and consists in these two Particulars. First, Plotting of Ground, or taking the Figure of it: Secondly, the Measuring of that Plot or Figure so taken, or taking the Superficial Content thereof.

1. The former of these, *viz.* The plotting of ground is excellently performed: First, By the Plain Table. Secondly, By the Theodolite: Or, Thirdly, By the Chain only, and Scale and Compass.

2. The latter, *viz.* The Measuring of a Plot is no more but the Reducing of it to one or more of the Three Regular Geometrical Figures before-mentioned, *viz.* either to a *Circle*, a *Square*, or a *Triangle*, (especially the last of these) and measuring it by their Rules, seeing no Plot or Figure can be so Irregular, but may be Reduced to Triangles, (if not to either of the former) and so may be measured by *Trigonometry*.

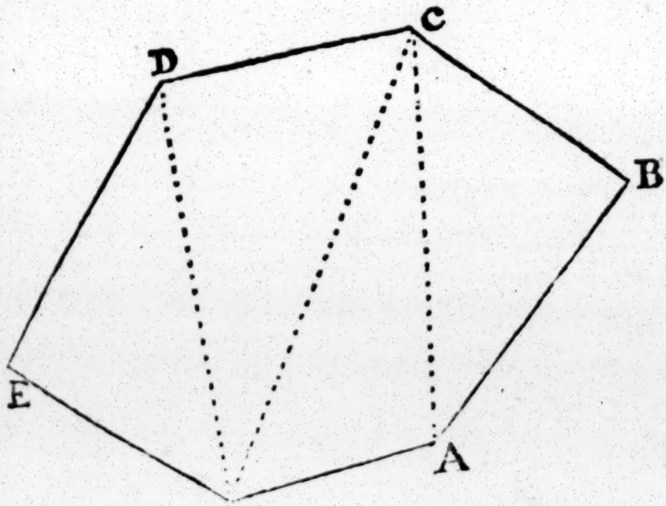
SECT. I.

First then, To take a Plot by the Plain Table.

A Plain Table is no more but a Board, or rather three pieces put together, making a long Square or Parallelogram, sufficient to hold a sheet of Paper, with Ledges

to

to keep the Table together, and the Paper faſt, with a looſe Index with Sights to lye upon the Table: The uſe is as follows: Firſt, Placing your Inſtrument upon the Staff, faſten it by the ſcrew in the ſocket, which it hath common with a Theodolite, and ſtanding at any Corner of the Incloſure, whoſe Plot you would take as at A, lay the Index with the ſights upon the Paper, and thorough the ſights eſpie ſuch a mark upon the Hedge or Side of a Field or Incloſure as far as the Hedge goes in a ſtreight Line as at B; and by the Fiducial



edge of the Index, draw a Line what length you pleaſe from A towards B: Then removing your Inſtrument meaſure with your Chain the diſtance betwixt A and B, and

 from

ſrom a Scale of equal parts prick down in Perches the ſaid Diſtance from A to B upon the Line drawn ; then placing your Inſtrument at B, lay the Index upon the Line A B, and turn the whole Inſtrument till through the ſights you eſpie A your firſt ſtation; there ſcrew the Inſtrument faſt upon the Staff, obſerving that A lay even with the Line drawn, then moving the Index to the Point B, keep one end thereof to that point; and move the other till through the ſights you eſpie a third mark at C, (which again is as far as the Hedge or Side of the incloſure goes in a right Line:) And by the fiducial Edge of the Index draw another Line to the point B, and meaſuring the diſtance by your Chain betwixt B and C, prick the ſame diſtance upon the Line BC from the ſame ſcale of equal parts, and remove your Inſtrument to C, and ſo from Point to Point, till you have gone round the incloſure: So ſhall you have a perfect Plot of the whole Field upon your Paper; and having ſo, it is eaſily reduced to Triangles, as is taught *pag.* 102. and meaſured by its Rule, *pag.* 107. whereunto I refer the Reader for brevity.

SECT.

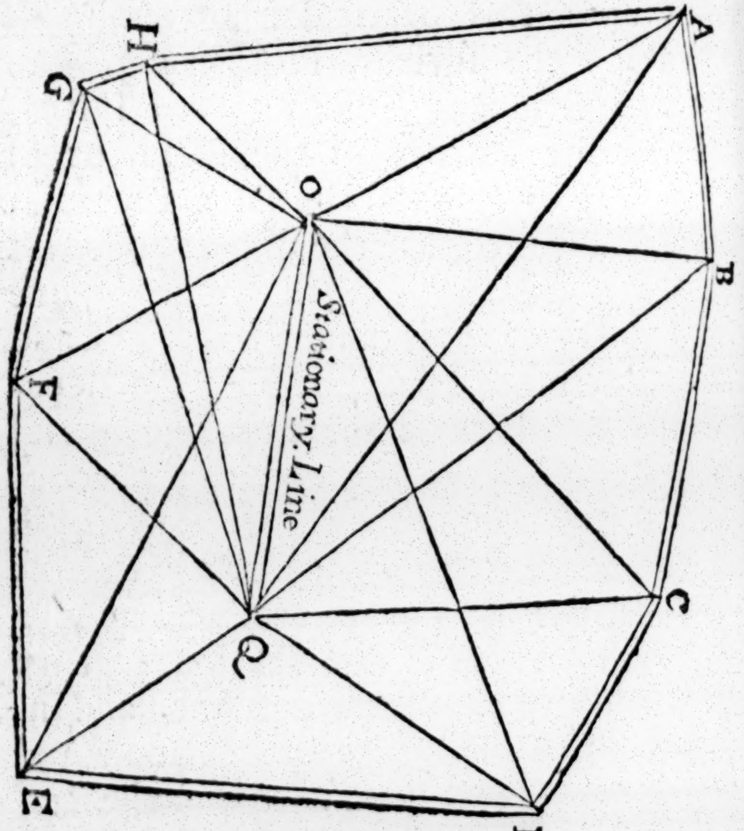
SECT. 2.

Secondly, To take a Plot by the Theodolite.

The Deſcription of a Theodolite you have *pag.* 183, and 194. The uſe of it in taking diſtances in general, you have alſo *pag.* 194, 195, 196. 197, 198; and as for meaſuring of Land thereby, it is no more but the particular application of thoſe generals upon this particular occaſion: As for taking the former Plot in *pag.* 223 by this Inſtrument, placing your Inſtrument in any Corner of the Field, or in any point of any of the ſides thereof; your buſineſs is to take the diſtance betwixt one point and another till you have gone round the Field, as is taught 194, 195, and 196: Therefore I ſhall not here uſe Repetition: But becauſe there is one ſpecial Example in *Leyburn*, how by the Theodolite to Plot a Field at two Stations taken in the middle thereof; from either of which all the Angles in the Field may be ſeen with meaſuring the Stationary Line only, being an excellent, pleaſant, and ingenious way, I ſhall here give it you: And for brevity ſake I ſhall acquaint the Reader, that it is the very ſame as is ſhewed *pag.* 200, and 201, KL being

 ſuppoſed

ſuppoſed the Stationary Line in the middle, and the whole Figure in *pag.* 120, being ſuppoſed to be only one half of the Plot, and Lines drawn from one point to another round the whole, as is there done; by which means you will have the Periphery of the whole Plot; but to give you an entire Example or Figure more, referring you to the former place for Direction, obſerve this following.



SECT.

SECT. 3.

Thirdly, To take the Plot of a Field without any other Instrument but the Chain only, and by your Scale to Protract the ſame upon Paper.

Let ABCDEF be a Field to be plotted, Firſt, Meaſure every ſide thereof with your Chain beginning at A, ſo ſhall you find,

The ſide	AB	contains 2 Chains	5 Links
	BC	1	48
	CD	2	6
	DE	4	75
	EF	2	15
	FA	2	52

Note that theſe are meaſured by Mr. *Gunters* Chain (which I do moſt commend, and which contains 4 Poles, each Pole 25 Links, or 100 Links in the whole Chain, each Link contains $7\frac{92}{100}$ inches, in all 792 Inches, 66 Foot, or 22 Yards.)

Now after that you have taken or meaſured theſe ſeveral ſides by this Chain, to Protract the ſame upon Paper, obſerve, That the Field contains 6 ſides, and therefore may be Reduced into two Trapezia's, or into 4 Triangles. A Trapezia is any Figure of 4 ſides, whoſe ſides and Angles are all unequal, ſo that this field may be redu-

ced

ced into the Trapezium ABCF, and FCED by a Line drawn from the Angle F, to the Angle C, which being meaſured by your Chain, you will find the Length of it to be 2 Chains 89 Links. Then again, if you meaſure with your Chain from C to A, which will contain 2 Chains, 70 Links, and from C to E, which contains 3 Chains, 60 Links; you will find the former two Trapezia's divided each into two Triangles, *viz.* the Trapezium ABCF into the Triangles ABC and ACF: And the other Trapezium FCED into the Triangles FEC and CDE; in every which Triangle you have all the ſides given, by means whereof you may draw upon Paper the exact Figure of your Field by what Scale you pleaſe, (according to the Rule given, *pag.* 85.) and as it here follows.

Firſt, Taking into your conſideration the Scituation of your Field, draw upon your Paper the Line AC, containing 2 Chains, 70 Links of any ſcale: Then becauſe the ſide AB contains 2 Chains, 5 Links, open your Compaſſes to 2, 5, and placing one foot in A, with the other deſcribe the pricked Arch *a b.* Likewiſe take in your Compaſſes 1, 48 the length of BC, and placing one Foot of your Compaſſes in C, with the other deſcribe the pricked Arch *c d*, croſſing the former Arch *ab* in the

the point B, then drawing 2 Lines from C and A to the point B, you have protracted the Triangle ABC: After the same manner must you protract the Triangles AFC, FEC, and CED, so is your work finished, and the Figure of the Field protracted upon Paper.

CHAP. VI.

Now for to know how to Measure this Plot thus protracted, or any of the 2 former, pag. 223, 224, *and by consequence any other: Take these Rules following.*

SECT. I.

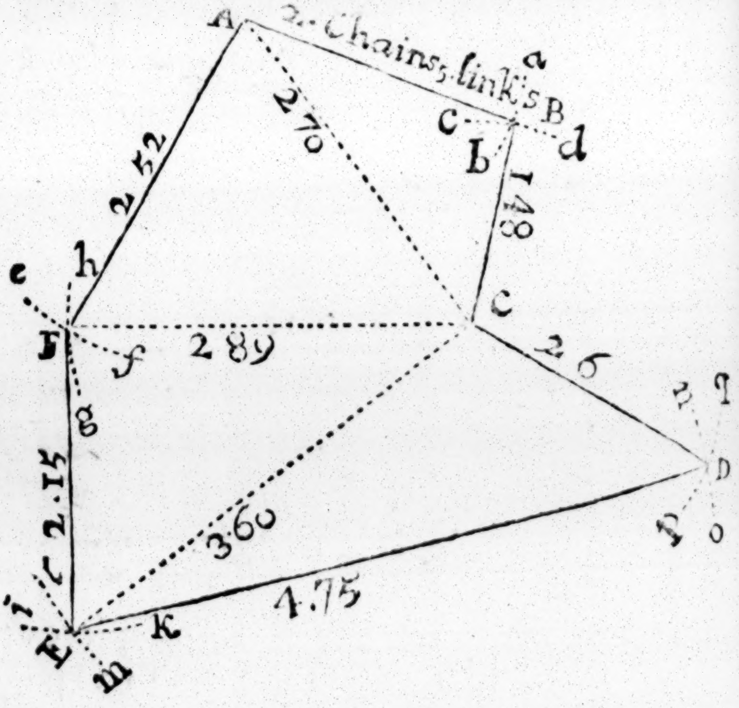
First, You must Reduce the said Plot or Figure into one or more of the Three Regular Geometrical Figures before-mentioned, viz. *the* Circle, Square, *or* Triangle.

NOW there are very few, or no pieces of Land which can be reduced to the first, *viz.* the *Circle*, but what may be done as to that you have *pag.* 74: But to the second, *viz.* the *Square*, many sorts of

of Figures may be reduced thereunto, as you may ſee *pag.* 89, 90. But in the third and laſt place, any Irregular Plot whatſoever may be reduced to Triangles, as you may ſee *pag.* 104 and 105, and as you ſee this Plot is done.

SECT. 2.

Now this being done, there is no more to be done, but to meaſure theſe ſeveral Triangles by letting fall perpendiculars, as



is taught *pag.* 55. and multiplying half the Baſe, and the whole Perpendicular, or the whole

whole Base and half the Perpendicular together, which will give the superficial Content of each Triangle, and the Contents of the several Triangles added together, will give the Content of the whole Plot in Acres, Roods, and Perches.

RULE 1.

And for finding the Superficial Content of a Triangle, having all the Three sides thereof given, it is taught *pag.* 113 which may be done without letting fall any perpendicular: For the doing whereof I refer the Reader to the place before-mentioned, *pag.* 113.

RULE 2.

The Second Rule is, To find how many Acres, Roods and Perches are contained in any Triangle, or any other piece of ground of any other shape. And for this purpose note, that 3 Barley Corns is an inch, 12 inches a Foot, 3 Foot a Yard, &c. And that by the Statute 5½ Yards, or 16½ Foot make a Perch or Pole, 40 square Perches make a Rood, and 4 Rood an Acre, so that 160 square Perches are an Acre: Suppose a piece of Ground should contain 917 square Perches, Divide 917 by 40, the Quotient

will

will be $22\frac{37}{40}$, divide this again by 4, the Quotient is 5, 2, 37, which is 5 Acres, 2 Rood, 37 Perches; or divide 917 by 160, the Quotient will be 5 Acres, and 117 Perches, which is 2 Rood, 37 Perches, as before.

RULE 3.

Note again to bring Chains and Links into Acres, Roods, and Perches: That 100 Links (of Mr. *Gunters*) make a Chain, 10 Square chains an Acre: Now ſuppoſe a Piece of Land to contain 9 Chains, 50 Links in length, and 6 Chains 25 Links in breadth, lying in a long Square: Multiply the one by the other, or 950 Links by 625, the Product is 5|93750, from which Product in Square Links cut alwaies off the five laſt Figures, ſo is the reſt to the left hand compleat Acres, which is here 5. Thoſe which are cut off being 93750 multiplyed by 4, (the number of Roods in one Acre) produceth 3|75000, from which Product cutting off again the 5 laſt Figures towards the Right Hand, the Remainder towards the Left Hand ſhews the compleat Roodes, which are here 3: The 5 Figures cut off being 75000, multiplyed by 40 the Number of Perches in a Rood, produceth 30|00000; ſo cutting off the 5 laſt Cyphers towards the

the Right hand, leaves 30 towards the Left, which are 30 Perches: If any odd Numbers had remained in those which were cut off, they had been so many Hundred Thousand parts of a Perch; but no Fraction remaining, the Contents of this Parallelogram or Long Square, is 5 Acres, 3 Roods, and 30 Perches: the like is to be done in any other.

SECT. 3.

But to help you herein, observe the Table following as to the former Example 593750. Cutting off the 5 last Figures, there remains 5 Acres: Then look for 90000 in the Table under the Title Links, and against it under the Title Roods and Perches, you find 3 Rood, 24 Perches: Then look for 3750, and against it you find 6 Perches, all which added together, gives you the Area or Contents of the whole, as before; 5 Acres, 3 Rood, and 30 Perches. The like is to be observed in any other Number of Square Links, to Reduce them to Acres, Roods, and Perches.

	A.	R.	P.
	5.	0	0
	——	3	24
	——	::	6
Somma	5	3	30

Square

Square Links.	R.	P.
100000	4	00
,90000	3	24
80000	3	08
70000	2	32
60000	2	16
50000	2	00
40000	1	24
30000	1	8
20000	0	32
10000	0	16
.9375	—	15
8750	—	14
8125	—	13
7500	—	12
6875	—	11
6250	—	10

SquareL.	R.	P.
5625	—	9
5000	—	8
4375	—	7
3750	—	6
3125	—	5
2500	—	4
1875	—	3
1250	—	2
.625	—	1

Note that 220 yds or 11 Score in Length, and 22 yds in Breadth makes an Acre.

SECT. 4.

Having the Length of any piece of ground in Perches, To find how many Perches in Breadth will make an Acre.

As the Length in Perches (which suppose 50) is to 160 upon the Line of Numbers, the same extent the same way will reach from 1 to the Breadth in Perches in this Example, $3\frac{20}{100}$, or $3\frac{1}{5}$ part of a Perch: *Di-*vide

vide 160 by 50, Quotient $3\frac{10}{50}$, which is the ſame. But in Chains and Links: As the length in Chains is to 10, ſo is one Acre to the Breadth in Chains and Links: As the Extent from 12 Chains 50 Links to 10, will reach from 100 Links, or 1 Acre to 80 Links the Breadth required. *Leyburn pag.* 276.

SECT. 5.

To know whether you have taken the Angles of a Field truly.

Add all the Angles together, and if the Sum of them all do agree with the Sum of 180 degrees multiplyed by a Number leſs by two than the Number of Angles in the Field, your work is true, otherwiſe not: This is where all the Angles are inward, but if any of the Angles be outward, then take the Complements of ſuch Angles to 180 Degrees, which is to be added to the reſt of the Angles inſtead of the Angles themſelves, and theſe outward Angles (if never ſo many) muſt not be accounted as Angles in the Multiplication, but wholly rejected.

To

To Reduce Chains and Links (of Gunters *Chain) into Feet and Yards.*

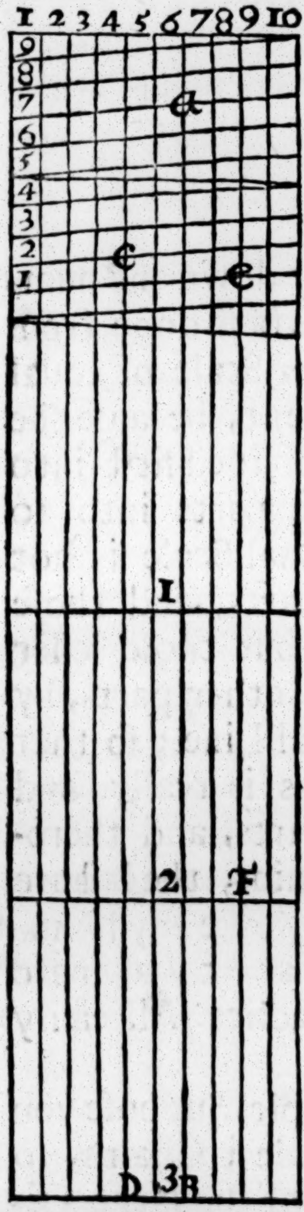
Set down the Number of Chains and Links as one whole Number, (as ſuppoſe 5 Chains 32 Links, ſet them down 5.32) multiply this 532 by 66, the Product is 351|12, from whence cut the two laſt Figures towards the Right hand, ſo ſhall the Figures to the Left hand remaining be Feet, and the Figures cut off 100 parts of Feet, *viz.* 351$\frac{12}{100}$ Feet, Theſe divided by 3, brings them into yards, *viz.* 117 yards. Or multiply 532 by 22, is 117|04, and cut off the two laſt Figures, brings it into yards at once, *viz.* 117 as before, and $\frac{4}{100}$ parts of a yard the Fraction. But let the Number of Chains be what they will, if the number of Links be leſs than 10, you muſt place a Cypher before the number of Links, as ſuppoſe 9 Chains 5 Links, place them thus, 9|05, then multiply them by 66, as in the other Example, will produce 59730, from which cut off the two laſt Figures, and there will appear to be 597 Feet $\frac{30}{100}$ parts, and ſo of any other: As for Example.

$$\begin{array}{r} 9.05 \\ 66 \\ \hline 5430 \\ 5430 \\ \hline 597.30 \end{array}$$

Note further, that to lay down any num-
er of Chains and Links, is beſt done from
Diagonal Scale, any other Scale of equal
arts not being ſo convenient, becauſe the
rdinary Scales are only divided firſt into
o parts, and one of thoſe parts into 10
ſſer parts; but the Diagonal Scale is not
nly firſt divided into 10 parts, and thoſe
gain into 10 leſſer parts; but thoſe leſſer
arts alſo divided into 10 other parts, by
eans of the croſs Diagonal Line; ſo that
ach of the largeſt Diviſions is really and
iſibly divided into 100 parts, and there-
ore taking the firſt for Chains, the laſt are
inks, and out of it you may take any Num-
er meaſured by your Chain to a ſingle
ink, to which a little practice will eaſily
ring you.

As to give you one Example, Suppoſe you
ere to take from your Scale 3 Chains, 76
inks. Firſt, Take three of the large Di-
iſions in your Compaſſes for the three
Chains:

Chains: Then for the Links obſerve, tha the ſmall Diagonal Diviſions are numbre from 1 to 10 upon the Top, and likewiſ



from 1 to 10 upon th ſide, now reckon upo the ſide upwards from to 7, and thoſe ſignifi Tens, and run along tha Line till you come juſt un der 6 on the Top, an thoſe ſignifie Units, an where thoſe two Line meet, there is the plac ſignifying 76. Therefor the Extent of your Com paſſes from *a* to B is ex actly 3 Chains, 76 Links and from D to C is Chains, 24 Links, an from *e* to F is 2 Chains 18 Links; and ſo of any o ther number that you have occaſion to protract, b the ſame more or leſs The Rule is the ſame al waies, counting upon th ſide upwards for Tens, an from the left ſide toward the right you muſt reckon Unites, and obſerving to meaſure Chains and Links alwaies upon the

ſame

me parallel Line, which is drawn from
e end of the Scale to the other.

SECT. 6.

o take the Plot of any Champian Field of 2000, *or* 3000 *Acres by the Plain Table, and never change Paper.*

Place your Instrument at every Angle, d ſo get every Angle and his ſides, not rearding the length of the containing ſides you be wont. Then Meaſure every edge, and as you uſe to lay the ſame down y your Scale and Compaſs, here you ſhall ut write the length of every Hedge upon e Lines drawn upon your Paper, ſo they eed not run off the ſame, becauſe you may raw them as long and as ſhort as you pleaſe, ut when you come at home, you muſt proract them according to due proportion.

SECT. 7.

Angles taken by the Interſection of Lines often fall out ſo acute, that it is hard to find the true point of Interſection.

Therefore when you have taken a Field t two Stations by interſection of Lines (as s taught, *pag.* 223.) Then from the firſt

Sta-

Station beyond the ſecond, or from the ſecond Station beyond the firſt, obſerve ſome Mark or Tree inclining towards the propoſed Field (which let make rather a Right Angle than obtuſe Angle with your Stationary Line) and ſo get the Angles it makes with both your Stations, making your Stationary Line one ſide of the ſaid Angles: Then go to the mark eſpied for a third Station, and there obſerve ſuch Corners that you thought would fall out Acute by Sections of Lines iſſuing from the firſt ſtations, and thereby get the Angles they make with your firſt and ſecond ſtations: for by help thereof, you ſhall correct the acute Section of the former Lines

And note, that where a Field lies ſo that you cannot from 2 Stations ſee all the Corners thereof, what you cannot do at the firſt and ſecond ſtation, you may do with as many more ſtations as need requires, as a third, fourth, or fifth Station, or more, as you ſee cauſe.

SECT. 8.

To take the Plot of a Wood, or Wateriſh Ground, when by Reaſon of ſome obſtruction or other, you cannot come within it.

Your way is to go round about the ſame, and

and make your obſervations at every Angle, the Diſtance between Angle and Angle meaſured by your Chain will be the ſides of the Plot, and protracting both the Angles and Sides upon Paper, will give you the true Figure of the whole, which you may meaſure by the Rules aforegoing.

SECT. 9.

Take notice, that the moſt, or however, very many ſmall incloſures are Trapezia, that is, having 4 unequal Sides, and 4 unequal Angles: Now any of theſe may be eaſily reduced into two Triangles, and ſo meaſured as I have formerly ſhewed you: But for the more ready caſting up the Contents of theſe, note, that the Diagonal Line drawn from Corner to Corner, whereby the Trapezia is divided into 2 Triangles, is by that means the common Baſe to them both. Therefore add the two perpendiculars together, the half of which multiplyed by the whole Baſe, the Product will ſhew the Contents of the whole Trapezia; or if you add both the perpendiculars together, and by that multiply the whole Baſe; half of the Product will be the Content of the Trapezia.

SECT. 10.

Another way to meaſure a Field of 4

unequal

unequal ſides is this: Firſt, Meaſure the top and the bottom, and add thoſe two together, and take the half, then meaſure the two oppoſite ſides, adding them alſo together, and take the half, Theſe two halfs multiplyed the one by the other, the Product will give you the true Contents: As ſuppoſe one ſide be 16 Perches, its oppoſite ſide 6, theſe added, makes 22, the half whereof is 11: Then ſuppoſe one of the other ſides 28, its oppoſite ſide 18, theſe added is 46, the half whereof is 23, which multiplyed by 11, ſhews 253 to be the whole Content. Note, that to protract a Trapezia, you muſt by the Theodolite, or ſome ſuch like Inſtrument obſerve one of the Angles, which being known and protracted, prick off the 2 containing ſides, and from the end or the Extremities of thoſe 2 ſides, you may draw the other two ſides by 2 Arches, croſſing one another at their Reſpective Diſtances, as in a Square or Triangle, as is elſewhere taught.

A Table

A TABLE of *FRACTIONS.*

O R,

Parts of Integer Pearches, that is, in any Number of Pearches, so far as the Table Extends. What a Quarter of a Pearch awanting, or over, will amount unto in Dayworks, Pearches, Yards, Foot, and Inches Square.

	P.		D.	P.	Y.	F.	I.		P.	D.	P.	Y.	F.	I.
A Quarter of a Pearch in Pearches.	1		0	0	1	1	1¼	A Quarter of a Pearch in Pearches.	21	1	1	1	1	1½
	2		0	0	2	2	3		22	1	1	2	2	3
	3		0	0	4	0	4½		23	1	1	4	0	4½
	4		0	1	0	0	0		24	1	2	0	0	0
	5		0	1	1	1	1¼		25	1	2	1	1	1
	6		0	1	2	2	3		26	1	2	2	2	3
	7		0	1	4	0	4½		27	1	2	4	0	4½
	8		0	2	0	0	0		28	1	3	0	0	0
	9		0	2	1	1	1¼		29	1	3	1	1	1½
	10	is	0	2	2	2	3		30	1	3	2	2	3
	11		0	2	4	0	4½		31	1	3	4	0	4½
	12		0	3	0	0	0		32	2	0	0	0	0
	13		0	3	1	1	1½		33	2	0	1	1	1½
	14		0	3	2	2	3		34	2	0	2	2	3
	15		0	3	4	0	4½		35	2	0	4	0	4½
	16		1	0	0	0	0		36	2	1	0	0	0
	17		1	0	1	1	1¼		37	2	1	1	1	1½
	18		1	0	2	2	3		38	2	1	2	2	3
	19		1	0	4	0	4½		39	2	1	4	0	4½
	20		1	1	0	0	0		40	2	2	0	0	0

 A Quar-

	P.		D.	P.	Y.	F.	I.		P.	D.	P.	Y.	F.	I.
A Quarter of a Pearch ; in Pearches.	41		2	2	1	1	1⅓	A Quarter of a Pearch in Pearches.	61	3	3	1	1	1½
	42		2	2	2	2	3		62	3	3	2	2	3
	43		2	2	4	0	4½		63	3	3	4	0	4½
	44		2	3	0	0	0		64	4	0	0	0	0
	45		2	3	1	1	1½		65	4	0	1	1	1½
	46		2	3	2	2	3		66	4	0	2	2	3
	47		2	3	4	0	4½		67	4	0	4	0	4½
	48		3	0	0	0	0		68	4	1	0	0	0
	49		3	0	1	1	1½		69	4	1	1	1	1½
	50	is	3	0	2	2	3		70	4	1	2	2	3
	51		3	0	4	0	4½		71	4	1	4	0	4½
	52		3	1	0	0	0		72	4	2	0	0	0
	53		3	1	1	1	1½		73	4	2	1	1	1½
	54		3	1	2	2	3		74	4	2	2	2	3
	55		3	1	4	0	4½		75	4	2	4	0	4½
	56		3	2	0	0	0		76	4	3	0	0	0
	57		3	2	1	1	1½		77	4	3	1	1	1½
	58		3	2	2	2	3		78	4	3	2	2	3
	59		3	2	4	0	4½		79	4	3	4	0	4½
	60		3	3	0	0	0		80	5	0	0	0	0

It you have occaſion for half Pearches, take theſe Sums twice ; if for three quarters take the ſame Sums thrice, and the total will be your deſire.

Note that 3 Barly Corns is an Inch, 12 inches a Foot, and 3 Foot a yard, 5½ Yards a Pearch, 4 Pearches a Day-work, 10 Day-work a Rood, 4 Rood or 40 Pearches an Acre: This

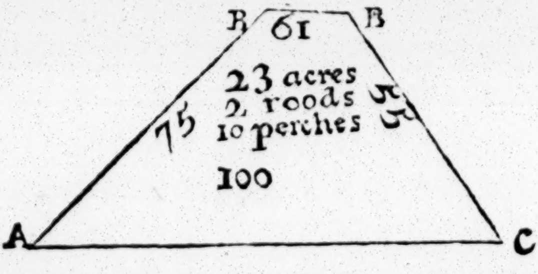
This Table may be very useful for those that have their Chains and Measuring Lines divided only into Pearches, half Pearches, and Quarters: But the most exact way is, to cast up by Chains and Links for such as have Mr. *Gunters* Chain, which way you have taught, *pag.* 231, 232. But I leave every one to use which way pleases him best, and so conclude *Surveying*.

As an Appendix I shall add these 3 Propositions following.

PROP. I.

How to take the Superficial Contents of Hills and Valleys according to Mr. Diggs.

Suppose ABC were a Mountain, first, measure the Circuit of the Base AC 100 Pearches: Then measure the Compass of the Summity or Top BB 16 pearches,



which added together, makes 116 pearches: Then measure the ascent of the Hill AB 75

 pearches,

pearches, and its oppoſite Aſcent C B 55 pearches, adding theſe two Aſcents together 130, half of which is 65, which multiplyed by Half of the Circuit 58, the Content will be 23 Acres, 2 Roods, and 10 pearches the true content of the Hill.

For meaſuring a Valley inſtead of the Circuit at the Baſe in a Mountain, you muſt meaſure the Circuit of the top of the Valley, and inſtead of the Circuit of the top of the Mountain, you are to meaſure the Circuit of the depth of the Valley, and inſtead of the Aſcenſes of the Hill, meaſure the Deſcenſes of the Valley, which makes it an hollow hill turn'd upſide down, and to be meaſured by the Rule aforegoing.

PROP. 2.

To find the Horizontal Line of a Hill or Mountain.

This is no more but reſolving a Hill into 2 right Angled Triangles, and then to find the Baſe of both thoſe Triangles which is the Horizontal Line, or the Line on which the Hill ſtandeth. Now to do this, place your Quadrant at the foot of the Hill on one Side, and through the Sights eſpying the top of the Hill (or a mark ſet as high above it, as your Inſtrument ſtands above the ground) Obſerve what degrees are cut by the Thread upon the Limb of the

the Quadrant (ſuppoſe 47 *d.*) then the Angle at the top muſt be the Complement thereof to 90, *viz.* 43 *d.* then meaſure the diſtance betwixt the top and the bottom, which ſuppoſe 71 feet, being the Hypothenuſal of the Triangle oppoſite to the right Angle 90, ſo having theſe given, you may eaſily find the Baſe and the perpendicular alſo, if required: For as 90 Degrees is to 71 Feet, ſo is 47 *d.* to the perpendicular, or 43 to the Baſe required. Now having got this part of the Horizontal Line, you muſt get the other part by proceeding after the ſame manner from the other ſide of the Hill, taking the Angles, meaſuring the Aſcent, and finding the Baſe as above, and theſe two Baſes added together, makes up the true Horizontal Line of the Hill or Mountain.

PROP. 3.

To find the Level betwixt any two places, and whether Water may be conveyed from a Spring-head, to any appointed place.

There is an Inſtrument called a Water-Level whereby this is beſt effected, though it may be alſo done by a Quadrant: For the Water-Level it is only a ſmall Pipe of Glaſs made cloſe at both ends with Wax,

and filled with Water, excepting a very small vacuity, which will shew it self in the middle, when it lies Level; but upon the least leaning either way, you will find it move accordingly. Now this laid in a piece of Wood, having sights at either end, is a fit Instrument for this purpose: This Water-Level placed at any convenient distance from the Spring-head in a right Line towards the place to which the Water is to be conveyed as 50 or 100 yards, and having two long Staves, cause one of the Staves to be set up at the Spring head, the other let be erected as many yards beyond your Instrument towards the place to which the Water is to be conveyed: These Station Staves erected perpendicular, and your Water-Level precisely horizontal, look through the Sightes to the first staff, causing a Leaf of Paper to be run up and down the Staff till it be espied through the Sights, there observe how many Feet, Inches, or Parts the Paper resteth upon, which note in your Book: Your Water-Level remaining immoveable, go to the other end of it, and causing a paper to be moved up and down the other Staff as before, note how many Feet, Inches, and Parts the paper resteth upon: Then if the second staff be more than the first, deduct the less from the greater, and the Remainder will shew

how much the ſecond ſtaff is lower than the firſt, and ſo by making ſeveral Stations you may find your deſire at what diſtance you pleaſe.

PROP. 4.

Another way by the Quadrant is this, Firſt, Go to the place whither you would convey your Water, and let any perſon ſtand toward the Spring-head any Number of Yards, and obſerve the top of a ſtaff in his hand equal to your eye by the Quadrant: Note the Degrees, and ſo take the Height as before at ſeveral other Diſtances, till you come to the Spring-head it ſelf, and protracting the ſeveral Angles of Altitude as you took your Stations and your diſtance of yards upon the Ground betwixt Station and Station: And in concluſion, you ſhall have the difference of Height betwixt the Spring-head, and the place whereunto you deſire to have your Water conveyed. This is of excellent uſe to find whether Coals may be ſaugh'd or drain'd of Water. Firſt, Boaring, or otherwiſe finding the Depth of the Coal from the ſurface of the Ground, and ſubſtracting that Height from the height found betwixt the place above ground where the Coals lye, and the place whither you deſire to bring your Water: All Coal be-

ing obſerved to lay true Level, let the ſurface of the Ground riſe or fall as it will, excepting where it is extraordinarily thrown up or down by that which they call an Horſe. This therefore may be done either of theſe waies aforementioned, that is, either by the Water-Level, or Quadrant, only obſerve, that at every Miles end there ought to be allowed $4\frac{1}{2}$ Inches more than the ſtrait Level for the Currant of the Water, which in a ſhort diſtance is not conſiderable.

OF

OF NAVIGATION.

CHAP. VII.

HAving now finished what I intend to say concerning *Surveying*, and taking of Heights and Distances at Land I come now to treat of that other part of *Geometry*, which is, the taking of Heights and Distances at Sea, to wit, *Navigation*: And I hope this Division of Geometry into *Surveying* and *Navigation*, and the Definition of them to be taking of Heights and Distances, the one at Land, and the other at Sea; may not by the Candid Reader be thought incongruous or improper, but proper and Genuine to the present purpose for making them plain and obvious to the Capacity of the meanest; which is the principal aim and design of this small Treatise. But to proceed, the whole Science of Astronomy,

nomy is a great Ornament and Accomplishment to him that professes *Navigation*; but of absolute Necessity *Navigation* must borrow of *Astronomy* the taking of Heights, *viz.* in these two particulars, (that is) First, The taking of the Height, or the Meridian Altitude of the Sun: And thereby in the second place, to find the Height or Elevation of the Pole: Yet this latter, in the same sense that it is called Latitude, may be the finding or taking of a Distance in *Navigation*, being indeed no more but the Distance *North* or *South* from the Equinoctial, or any other supposed parallel; as Longitude is the Distance *East* and *West* from the first Meridian, or any other supposed Meridian: And as the Rumb leading from one place to another may be called the Distance run upon such a point of the Compass. Now the taking these Heights and Distances is the Sum and Substance of the Art of *Navigation*. And to Treat of these particulars in order, shall be the method of my intended Discourse upon this Subject.

Now these may be more succinctly comprized in these four Terms, being the same which are used by *Norwood*, *Hues*, and others. To wit,

1. The

1. The Difference of Latitude.
2. The difference of Longitude.
3. The Rumbs.
4. The Diſtance run upon the ſaid Rumb.

Any two of theſe being known or given, the other two may alſo be found; whence obſerve, That there is a two-fold finding of each of theſe. Firſt, As they are *Data*, to be given or known, and ſo they are found by obſervation. Secondly, As they are *Quæſita*, ſought or required, and ſo they are found by *Trigonometry*, or *Arithmetical Calculation* in the Reſolution of a Plain Right Angled Triangle; ſo that in conclusion you will find that *Navigation* as well as *Surveying* is ſtill no more but the Reſolution of a Triangle: But to proceed Methodically: Seeing there muſt be alwaies two of theſe given or known, to find the reſt: It will be very neceſſary to ſhew you how in the firſt place they may be found as *Data* by Obſervation.

SECT. I.

And first for finding the Latitude or the Elevation of the Pole.

PROBL. I.

This is done by observing the Meridian Altitude of the Sun or Stars either by an Astrolabe, Cross-staff, or Quadrant.

How to take the Meridian (or any other) Altitude of the Sun or Stars by the Cross-staff by forward Observation, is the same as in the General use of that Instrument in taking of Altitudes, as is taught *pag.* 212, 213: And how to take the same Altitude by Thread and Plummet, and backward observation, is also taught particularly *pag.* 212. §. 4. But the easiest and readiest way to take the Meridian, (or other) Altitude of the Sun is, to do it by the Astrolabe or Quadrant thus, by that which is called backward Observation, and without looking through the sights: Let the Sun shine through that sight which is next the Center, and bring that Beam to fall right upon the hole of the other sight, and the Thread will fall upon the true Altitude in the Quadrant, or in the same manner the Index

Index in the Aſtrolabe will cut the Degrees of Altitude.

Now to find the Meridian Altitude of the Sun, (though the Sun do not ſhine) may be done by knowing the Declination and Latitude.

For if the Declination be North, you muſt add it to the Complement of the Latitude, (which is alwaies the ſame with the Height of the Equinoctial ;) and if the Declination be South, you muſt ſubſtract it from the Complement of the Latitude, which will give you the Meridian Altitude at any time. As ſuppoſe at *London* the Elevation of the Pole (or the Latitude) is 52*d.* the Complement whereof to 90 *d.* is 38 *d.* (which is alſo the Height of the Equinoctial.) Now the ſecond of *May* the Sun being in 20 *d.* 42 *m.* of *Taurus*, his declination Northwards is 117 *d.* 56′. 21″. which added to 38, makes the Meridian Altitude of the Sun to be 55 Degrees, 56 Minutes, and 21 Seconds.

If it be at the time of the Equinoctial, (*viz.* 11*th* of *March*, or the 13*th* of *September*) the Height of the Sun or Star (when they are upon the Meridian) ſubſtracted from 90 Degrees, will ſhew the true Latitude ; but at any other time you muſt find out the Declination of the Sun or Stars, and

and it the Declination be Northerly, then substract it from the Altitude: If Southerly, you must add it to the Altitude; by which Addition or Substraction you shall have the Height of the Equinoctial above the Horizon, and that being substracted from 90 Degrees, will shew the true Latitude of the place where you then are.

PROBL. 2.

To find the Elevation of the Pole by the Globe.

First, Taking the Meridian Altitude of the Sun, and bring the place of the Sun in the *Ecliptick*, or the Star to the *Brazen Meridian*, and so move the *Brazen Meridian* together with the Globe through the Notches it stands in, till the place of the Sun, or the Star be elevated so many Degrees above the Horizon, as the Suns or Stars Meridian Altitude is: The Globe standing in this position, the Pole will be elevated according to the true Latitude of the place. *Example*,

The 12*th* of *June* I find the Suns place in the beginning of Cancer, and the Meridian Altitude of the Sun at *London* 62 degrees; bring the first degree of Cancer to the Meridian, and elevate the same 62 degrees above the Horizon, so you will find the Pole elevated at *London* 51 Degrees, 30 Minutes:

Minutes: Or by the former Rule, having the Suns Meridian Altitude 62, and finding by the Table of the Suns Declination or otherwise, that the 12*th* of *June* the Sun hath his greatest Declination, *viz.* 23 *d.* 30 *m.* substract this from 62 *d.* rests 38 *d.* 30 *m.* for the Height of the Equinoctial; and that again substracted from 90 *d.* leaves 51. 30 for the Latitude or Elevation of the Pole.

And seeing the finding of the Latitude by the Sun depends upon knowing his place in the Zodiack, or his Declination; and if you would find the Latitude by the stars, you must also know their Declinations. Therefore I shall give you two short Rules for the two former, and a short Table for the latter in this place, but must refer the Reader to larger Rules and Tables when I come to speak of them in Astronomy.

PROBL. 3.

To find the Suns place by these two Verses.

♈ ♉ ♊ ♋ ♌ ♍
Evil, attends, its, object, unvail'd, vice,
♎ ♏ ♐ ♑ ♒ ♓
Vain, Villains, jest, into, a, Paradise:

These 12 words signifie the 12 months beginning

ginning at *March*, and the 12 signs in orde
the Sun enters into every Moneth, and t
know what day of the Moneth he enters
If the first Letter of the word proper t
the Month be a Consonant as Paradise be-
longing to *February*, in that Moneth he en-
ters the 8*th* day, but if it be a Vowel, (as a
the rest are) add so many days to 8, as th
Vowel denotes in order of the 5 Vowel
a e i o u / 1 2 3 4 5 } which will shew what day of th
Moneth the *S*un enters any of the 12 signs
and reckoning that the sun goes thorough
one Degree every day, you may find th
Degree the sun is in, and (by the hours of
the day from Noon) the Minutes of tha
degree.

P R O B L. 4.

To find the Suns Declination by Logarithmes, or the Lines of Artificial Sines.

As Radius or 90 Degrees is to the *Suns* greatest Declination (which is alwaies 23 Degrees 30 Minutes) *S*o is (the Number of Degrees or Minutes from the next Equino-ctial point to the suns place) to the suns present Declination: *S*uppose the suns place be o degrees, or the beginning of *Taurus*, that is, 30 degrees from the next Equino-

ctial

ſtial point of Aries, his Declination will be found to be 11 *d.* 30 *m.* for as 90 to 23, 30, ſo 30 to 11, 30, the like Proportion will hold in any other.

PROBL. 5.

To find the Declination of ſome notable Stars, obſerve this Table.

North			South		
The firſt Horſe in the wain.	57	27	Spica Virginis.	8	55
The third in the wain.	51	5			
Arcturus.	21	53	Heart of Scorpion.	25	30
The Eagle.	7	57	Orions right ſhoulder.	6	16
The Bulls Eye.	15	54			
Caſtor.	32	30	Orions left foot.	9	10
Pollux.	28	30	The great Dog.	15	56
The Lyons Heart.	13	40	The little Dog.	5	53
Theſe ſtars have North Declination.			Theſe have South Declinations.		

Thus much for Latitude.

SECT. 2.

In the next place, for finding the Longitude.

If the Longitude could be as certainly found as the Latitude, it would render *Navigation* perfect; but notwithſtanding that many have pretended thereunto, yet none

none have attained to it in reality as far as I can yet find: *Theaker* in his Planisphere, which he calls a light to the Longitude, tells you he can by that Instrument as certainly do it, as to find the Rising, Setting, and Southing of the Stars, but if he can, he does very ill express it, and I must conclude, that, *aliquid latet quod non patet*, and perhaps he might keep a reserve to himself on purpose.

The most Rational waies of finding the Longitude which I meet with, are these 4, Collected out of *Varenius*, and *Carpenters* Geography.

PROBL. 1.

First, By an Eclipse of the Moon.

By observing how much sooner the eclipse beginneth at a place of known Longitude, (which you may find by an Ephemerides) than at the place where you are; where you may find the time of the Eclipse by observation with the Telescope, and knowing the Latitude at the same time by the Stars, you may find the true hour of the Night. This done, the difference of this time of the Moons entrance into the Eclipse, or the middle or ending of the same at the place where your observation was made, being

being converted into Degrees and Minutes, and either added or ſubſtracted from the hour of the beginning, middle, or end of the Eclipſe at the other place of known Longitude found by the Ephemerides : I ſay the Degrees and Minutes of the difference betwixt the hour at one place, and the hour at the other place added or ſubſtracted from the Degrees and Minutes of the known Longitude, gives you the Longitude of the place required.

PROBL. 2.

By a Clock, Watch, or Hour-glaſs.

If you have a very true Watch, and be to ſail from a place of known Longitude, when you depart from that place, by an Aſtrolabe, or otherwiſe by the Sun or Stars obſerve the hour of the day or night, and ſet your Watch exactly to the ſame hour, when you have a mind to know what Longitude you are in, obſerve again by the Aſtrolabe, or otherwiſe the exact hour ; and if the Watch agree with the obſervation, there is no difference of Longitude ; or what difference betwixt theſe two you find added or ſubſtracted to or from the Longitude of the place you came from, ſhews the Longitude of the place you are now

now at: This way you see depends upon the exact taking the hour at both places, and upon the exactness of the Watch; therefore in this latter respect an Hour-glass is accounted fitter for Sea.

PROBL. 3.

Thirdly, By observation of the Difference in the Sun and Moons motion.

First, You must take for granted, that the Motion of the Moon is 48 minutes of an hour slower in 24 Hours, or 360 Degrees than that of the Sun. Secondly, You must suppose that by Mathematical helps a man may know in any place the Meridian, (or South point) and also the hour of the day, and at least by an Ephemerides the time of the Moons coming to the South, or to the Meridian. Now suppose at *London* you find (by an Ephemerides calculated for that place) that the Moon comes to the Meridian (on some set day) at four a Clock after Noon: And you being in the *West-Indies* the same day you observe the Moon come to the Meridian at 10 Minutes after four: Then say by the Rule of Proportion, If the difference of the Sun and Moons motion be 48 Minutes of an hour in 360 Degrees; what will it be in 10 Minutes: Or if 48 give

360

360, what gives 10? The fourth proportional Number will be 75, which is the distance of that place from *London*, from which number the Longitude of *London* being substracted (20 degrees) and the Remainder (55) again substracted from 360, the Residue will shew the Longitude (305).

P R O B *L.* 4.

By the Distance betwixt the Moon, and some known Star scituate near the Eclipticks.

But this, and several other waies mentioned in *Varenius* as by the Moons place, her entrance into the Ecliptick, *& per Planetas Joviales:* I shall not trouble the Reader with, but refer him to the Author, 418, 420, 421, and pass on. Now the Difference of Latitude, or the difference of Longitude, is so many Degrees, Minutes, or Miles as the Latitude or Longitude of one place differs from the Latitude or Longitude of another place, the Latitude or Longitude being observed or known at both places; as knowing the Latitude of the place from whence you set Sail, and having sailed a certain number of Leagues, there you take an observation of the Latitude as is beforementioned: Now if both the places be on the *North-side* the Equinoctial, or on

on the ſouthſide the ſame ſubſtract the leſſer latitude from the greater, the remainder is the difference of Latitude: If one of the places be under the Equinoctial, and the other on the North or South ſide thereof the difference of Latitude is the Latitude itſelf. If one of the places be on this ſide the Equinoctial, and the other on the other ſide, the Sum of both the Latitudes added together is the difference of Latitude, or the Diſtance North and South betwixt the two places; and if the Longitude could be as exactly found, it would ſhew their Diſtance Eaſt and Weſt: However the difference of Longitude is the Diſtance of any place upon the Line of Eaſt and Weſt from the Meridian of any other place propoſed, but becauſe this cannot certainly be found or however not without much difficulty by obſervation it is ſeldom given, but required in *Trigonometry*, and found by the Reſolution of a Triangle, as ſhall be ſhewed hereafter in its due place, and ſo much for the two former particulars, the Difference of Latitude and Longitude.

SECT. 3:

In the third place we come to ſpeak of the third conſiderable in Navigation, *viz. the* Rumb

Which in plain *Engliſh* is the point of the

the Compaſs upon which the ſhip Sails, and this is uſually one of the *Data's*, or things given, becauſe the Sea mans Compaſſe by which it is directed or ſteered alwaies ſhews this. The finding out of that moſt admirable vertue of the Magnet, being the greateſt advantage that ever hapned to *Navigation*: Indeed the difficulty lies in this, that the Needle touched by the Loadſtone in ſeveral places admits of ſeveral Variations, which muſt be carefully obſerved and taken notice of.

SECT. 4.

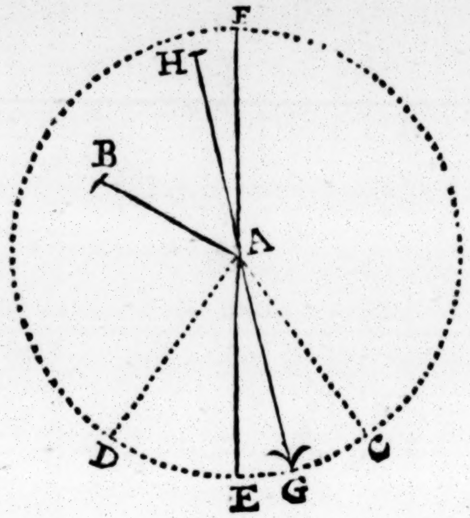
To find this Variation of the Needle or Compaſs.

There are ſeveral waies, As by the *Azimuth Compaſs*, and by the *Sea-Rings* deſcribed by Mr. *Wright*; but theſe are chargeable. I ſhall in this place only ſhew you how to do it theſe three waies, the one at Land, the other will ſerve both at Sea and Land.

1. *To find the Variation of the Compaſs, and withal, a true Meridian Line.*

Firſt, Deſcribe ſeveral Circles upon a Plain, and in the Centre place a *Gnomon* or Wire perpendicular as AB. In the Fore-

 noon



noon obſerve when the Extremity of the ſhadow of AB juſt touches one of the ſaid Circles, and make there a mark at C. In the Afternoon obſerve again, when the extremity of the ſhadow of AB toucheth the ſame Circle, making there another mark at D. Divide the Diſtance CD into 2 equal parts, which ſuppoſe at E draw the Line EAF, which is the Meridian Line: Then place a Needle GH upon the Center, and mark how many degrees the point of the Needle G is from E, ſo much doth the Needle vary from the North in that place.

II. *To find the Variation of the Needle by the Globe either at Sea or Land.*

Firſt, Rectifie the Globe, and bring the

Suns

Suns place to the East side of the Horizon, and look in the Circle of Winds, and against the place of the Sun you have the point of the Compass upon which the Sun riseth: Then observe by the Compass what point of the Compass the Sun riseth or sets on Morning or Evening, and see what difference there is by the Globe and by the Compass, and that is the Variation (if any be.) Allowing for this variation, the Needle will alwaies shew the Rumb, or the true point of the Compass upon which the Ship is steer'd.

III. *To find the Suns Amplitude, and thereby the Variation of the Compass.*

As the Co-sine of the Latitude is to Radius, so is the sine of the Declination to the sine of the Amplitude. Suppose the Latitude 51 *d.* 32 *m.* its Co-sine or Complement is 38 *d.* 28 *m.* the Suns Declination 15 *d.* 10 *m.* the Amplitude will be found to be 24 *d.* 52 *m.* North, because the Declination is North. Now the Circumference of the Compass divided into 360 *d.* observe at the Suns rising and setting how many degrees the Sun is from the direct point of the Amplitude, and so much doth the Needle varie in that place. This Observation is best to be made, when the lower edge

of the Sun ſeems juſt to touch the Horizon.

SECT. 5.

In the fifth and laſt place, To find the Diſtance Run.

This is to know what way the Ship mskes in ſuch a ſpace of time, or how many Leagues or Miles the Ship runs in an hour, *&c.* And this is uſually obſerved by the Log-Line and Minute-Glaſs; ſo many knots as the Ship runs in half a Minute, ſo many Miles ſhe saileth in an hour; the ſpace between every knot being $41\frac{2}{3}$ Feet, reckoning 12 Inches to a Foot, 5 Foot to a Pace, and 1000 Paces to an *Engliſh* Mile: But enlarging the Mile according to Mr. *Norwoods* Obſervation, the diſtance between every knot muſt be 50 Foot, and then as many of thoſe as run out in half a Minute, ſo many Miles or Minutes the Ship ſails in an hour, and for every Foot more the tenth part of a Mile: And if you pleaſe, you may bring theſe Miles into Leagues according to the uſual Cuſtom of Sea-men; and every time the Log is caſt, it ought to be noted upon the Log-board what way the Ship makes. Now beſides the Log-line, there hath been ſeveral other Engines invented, ſome with Wheels, and ſome other

other waies for this purpoſe; As I have ſeen a wheel to Meaſure Land withal, and may be applyed to know how many Miles a Coach goes in an hour, or the like. And for to meaſure any ſmall quantity of time, I find no better an Invention than that of a Swing (being proved by Experience) that a Bullet *&c.* hung in a ſtring, and being moved from the perpendicular to any Angle (keeping the ſtring ſtrait) and ſo let go to have its own Swing, will keep ſuch exact time in its ſwinging, that though it fetch a larger Compaſs at firſt than afterwards, yet it keeps the very ſame time in every Swing, only the length of the ſtring is the thing that alters the time; and therefore if you would have it go faſter or ſlower, you muſt make your ſtring ſhorter or longer: As a ſtring of $38\frac{1}{2}$ Inches long, (meaſuring from the Center of the Plummet) will exactly make 60 ſwings in a Minute of an hour, and ſo proportionably if you would have fewer ſwings in the time, you muſt lengthen the ſtring, or if more, you muſt ſhorten it. This is meant when the ſtring is faſtned to ſomething in the top of a Room, or ſo: But if you hold it in your hand, it is obſerved that $31\frac{1}{2}$ Inches in length will do the ſame as $38\frac{1}{2}$ faſtned to any ſteddy thing. This may very properly be uſed in ſtead of the half Minute glaſs aboard a Ship.

SECT. 6.

Thus you ſee how you may find by obſervation the Latitude and Longitude, the Rumb, and the Diſtance run: Any two of which being known (by reaſon of the Right Angle, which is alſo alwaies known:) The other two may be eaſily found by *Trigonometry*, or the Reſolution of a Plain Right

Angled Triangle, as you may ſee in the Triangle ABC: The Hypothenuſe alwaies repreſenting the Diſtance run AB, the Angle at the North point A repreſents the Rumb,

Rumb which (in the Triangle DAC is North-West and by North:) in the Triangle BAC is North-West: The Base BC is the difference of Longitude or Departure from the Meridian; the perpendicular AC is the Difference of Latitude, and the Angle at C you see is a Right Angle; and the Angle at B should be the complement of the Rumb.

Now the Transmutation of the things to be granted or known, and to be enquired after in these four Terms, (*viz.* the Difference of Latitude, the difference of Longitude, the Rumb, and the Distance run) may be proposed six manner of waies, and every Proposition resolved by the Resolution of this plain Right-Angled Triangle, as follows in Sailing by the *Plain Cart.*

PROP. 1.

The Difference of Latitude AC and Difference of Longitude BC being known, The Rumb and Distance *i. e.* the Angle A, and the Hypothenuse AB may be found.

This is the same as having two sides and a Right Angle betwixt them, to find the other two Angles, and the third side, as is taught *pag.* 137. Or by Protraction, thus:

 First,

First, lay down the Base 90 Miles, being the difference of Longitude BC, and upon the point C protract an Angle of 90 *d.* laying down the perpendicular 87 A C, being the difference of Latitude, and to touch the ends or extremities of both these draw the Hypothenuse or Distance run 126 A B, so shall the Angle at A be the Rumb 45 *d.* *vide pag.* 132.

PROP. 2.

The Difference of Longitude BC, and the Rumb A being known; The difference of Latitude, side AC, and the Distance run, *s.* AB. may be found.

This is the same, as to have 2 Angles and a side given, to find the other two sides: which is done by the general Rule, *pag.* 141. For as the Angle at A, which represents the Rumb 45 is to the Base BC 90 Miles, so is the Right Angle at C 90 *d.* to the Distance *s.* AB 126 Miles, and so is the Complement of the Rumb, *viz.* 45 at the Angle B to the perpendicular or difference of Latitude 87 Miles AC, either to be wrought by Logarithmes, *pag* 31. or by the Lines of Numbers, Sines, and Tangents,

gents, *pag*. 41. or by Protraction, *pag*. 132. to which places I refer the Reader.

PROP. 3.

The difference of Longitude BC, and the Distance run *s*. AB. being known.

The Difference of Latitude *s*. AC, and the Rumb Angle A, may be found.

Supposing the Right Angle at C 90 *d*. to be known with the other 2 sides BC and AB. This is the same, as having two sides and an Angle opposite to one of them, to find the other Angle opposite to the other side given, and the third side. Therefore by the Rule of 3 say, as 90, or Radius is to *s*. AB. 126, so is the *s*. BC. 90 Miles to the Angle A 45 *d*.

PROP. 4.

The Difference of the Latitude AC, and the Rumb, the Angle at A being known,

The difference of Longitude BC, and the Distance run *s* AB. may be found.

In this Proposition (supposing the Right Angle also given) you have all the three Angles given, the third Angle being the

Complement of the Rumb to 90, *viz.* 54 *d.* Therefore ſay by the Rule of Proportion, As the Complement of the Angle at A, *viz.* the Angle B 45 *d.* is to the *s.* AC, ſo is the Rumb or Angle at A 45 *d.* to the *s.* BC, and ſo is Radius or 90 *d.* to the *s.* AB.

PROP. 5.

The Difference of the Latitude AC, and the Diſtance run AB, with the Right Angle C being known,

The Rumb A, and Difference of Longitude B C, may be found.

This is to have two ſides and an Angle oppoſite to one of them given, to find the Reſt, as in Prop. 3. As the Angle C is to *s.* A B, ſo is *s.* AC to the Angle B, and ſo is its Complement A to *s.* BC, which Complement is the Rumb, and the ſide BC is the difference of Longitude required.

PROP. 6.

The Rumb, Angle at A, and the Diſtance run AB, with the Right Angle being known,

The Difference of Longitude BC, and Difference of Latitude AC. may be found.

This

This Propoſition is of all other the moſt uſeful, theſe being the moſt uſual *Data*'s to find the other; moſt Sea-men keeping an Account only of the Rumb or Point of the Compaſs ſail'd upon; and the Diſtance run by often caſting the Log-line.

And it is the ſame as having 2 Angles and a ſide oppoſite to one of them given, to find the reſt of the Triangle, as *prop.* 2, 3, 5, therefore by the Rule of Proportion, ſay, As the Angle C 90 *d.* is to *s.* AB the diſtance run, ſo is the Rumb or Angle A to the *s.* BC the Difference of Longitude, and ſo is the Complement of the Rumb to 90, *viz.* Angle B, to *s.* AC the difference of Latitude. In this Propoſition, as alſo in any other, where the Rumb is given, as *Prop.* 2, 4. having the Rumb given, and the Right Angle, you have alſo the third Angle given, being the Complement of the Rumb to 90, which makes it very eaſie to be reſolved. And thus you ſee theſe 6 Propoſitions of *Plain Sailing*, (or if there can be more Tranſmutations of theſe 4 principal Terms in *Navigation*) may all be reſolved by *Trigonometry*: So that hence you will clearly diſcern, that not only *Surveying*, but *Navigation* alſo is performed by the Reſolution of a Triangle, and that only a Plain Triangle; and Navigation (altogether) and *Surveying* for the moſt part, by a

Right

Right Angled Plain Triangle only: whereby the easineſs of both, and the excellency of *Trigonometry* do moſt manifeſtly appear.

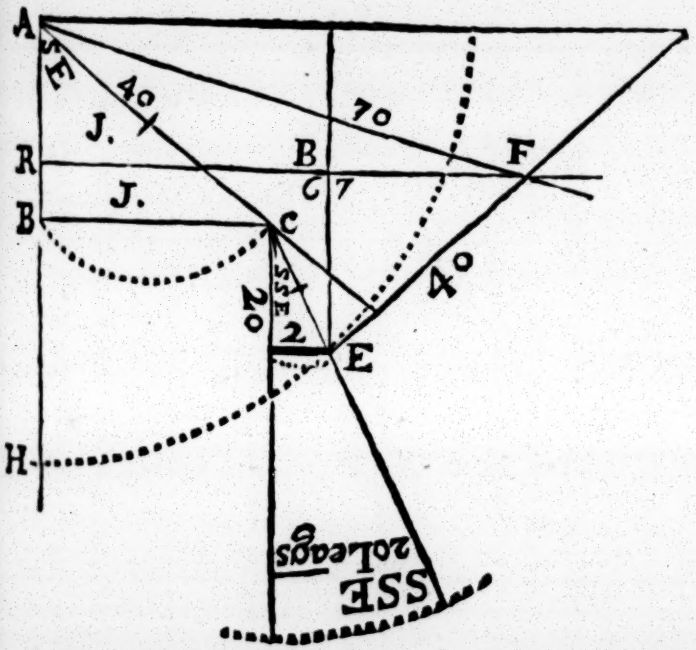
SECT. 6.

A Travis.

What hath hitherto been ſpoken concerning *Navigation*, reſpects only a direct ſingle Courſe where a man needs not Lavere, or change his courſe at all; and for a *Travis*, it is the ſame thing only wrought by ſo many more Triangles, every time you change your courſe conſtitutes a new Triangle, as by the following Example.

As ſuppoſe I ſet out from *Sommars Iſland*, lying in the Latitude of 32 *d.* 20 *m.* North Latitude, and Sail *S E* 40 Leagues; from thence I Sail *S S E*, till I alter my Latitude one Degree; and from thence upon ſome point between North and Eaſt I Sail 40 Leagues, till I alter my Latitude one Degree, 45 *m* I demand what Latitude I am in; my diſtance from the place whence I ſet Sail: My departure from my firſt Meridian, likewiſe my Courſe made good: When all this is done and protracted, the difference of Latitude is AR, reckoning 20 Leagues, or 60 Miles to a Degree, is 21 Leagues, or 63 Miles; and therefore in degrees.

grees, 1 *d.* 3 *m.* which becauſe it is Southerly, ſubſtract from your firſt Latitude, (if it had been Northerly, you muſt have added it thereunto.) And that is your preſent Latitude.



Secondly, The diſtance run (as if you had gone directly ſtreight) is AF 70 Leagues taken from the ſame Scale of equal parts, as AC or EF was taken.

Thirdly, The Departure from the Meridian is RF 67.

Fourthly, The point of the Compaſs that the Ship lies now from *Sommars Iſland*, is from H to F, *viz.* ESE half a Point Eaſterly.

PROBL.

PROBL. 1.

Sailing away *W S W*, I ſee a point of Land which I find to bear from me *W B N*, and having Sailed 6 *Leagues* further I find it bears from me *NWBW*, I would know how far it is Diſtant from me.

This is done in Plain Sailing after the ſame manner as to take an inacceſſible diſtance at 2 Stations at *Land*, as in *pag*. 197.

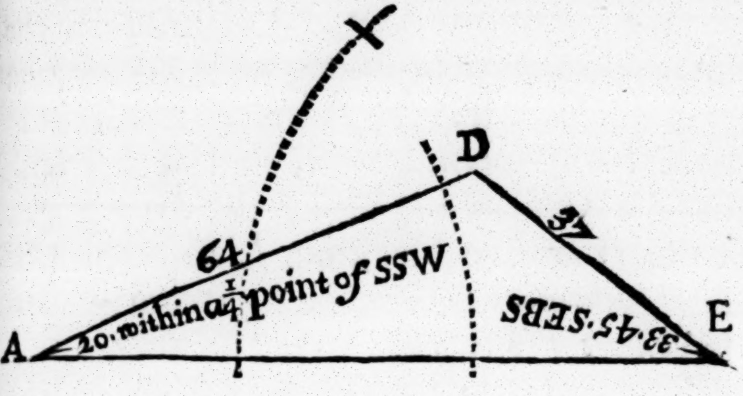
PROBL. 2.

A Merchant Man being in the *Latitude* of 43 *d.* falls into the hands of Pyrates, who amongſt other things take away his Sea-Compaſs, but when he is gotten clear, he ſails away as directly as he can, and after two days meets with a Man of War, who alſo had been the day before in the *Latitude* of 43 *d.* and had Sailed thence SEBS. 37 *Leagues.* He being deſirous to find thoſe Pyrates, the Merchant Man tells him he left them lying to and fro where they took him, and he had Sailed ſince at leaſt 64 *Leagues* between the South and Weſt, what Courſe ſhall the Man of War ſhape to find theſe Pirates.

This is done much after the ſame manner as the former, ſuppoſe AE the Parallel of

43 *d.*

43 *d.* D the place where the Ships meet, the Angle at E, the Rumb upon which the



Man of War Sailed SEBS 33 *d.* 45 *m.* DE his diſtance 37 *Leagues*, A D the diſtance Sail'd by the Merchant Man. Now by theſe you are to find the Angle A, which you will find 20 *d.* within 1/4 point of *SSW*, ſo hath the Merchant Man ſailed, and therefore the Man of War muſt Sail *NNE*, within 1/4 point of the Compaſs.

I might add many more ſuch like as you find them in *Norwoods* Epitomy, to which I refer the Reader; done after the manner of Plain Sailing, ſuppoſing the Degrees of *Longitude* and *Latitude* are both equal.

This *Travis*, as alſo the eight former Propoſitions are all wrought by that way of Sailing, which we call *Plain-Sailing*, or Sailing by the Plain Cart, which is very eaſie and alſo exact enough for ſhort Diſtances, and near the Equinoctial; in other Voyages it

it is ſubject to Error, as ſuppoſing the Degrees of Longitude to be the ſame in every parallel of Latitude; or that the Degrees both of Longitude and Latitude are the ſame all the world over, which is a grand Miſtake, as Mr. *Wright* makes it appear in his Errors of *Navigation*, and withal hath found a Remedy for it, by drawing a true Sea-Cart by the Meridian Line, which he calls *Mercators Chart*, being according to *Mercators Projection.*

The Difference between Sailing by the *Plain Chart*, and Sailing by *Mercators Chart*; that is, the Difference betwixt keeping an account either by the one, or by the other, conſiſts in theſe particulars.

Firſt, In the *Plain Chart* both the Meridians and *Parallels*, or (in plain Termes) the Degrees of Longitude and Latitude, are all equal, and meaſured by the ſame Scale of equal Parts, and therefore by conſequence the Diſtance muſt be alſo meaſured by the ſame Scale, and ſo all the three ſides of any Plain Right-Angled Triangle repreſenting theſe, *viz.* The Baſe, the Perpendicular, and the Hypothenuſe are all meaſured by one and the ſame Line of equal Parts,

But finding this way, *viz.* by the *Plain-Chart* very erroneous, being grounded upon errone-

erroneous principles, as is shewed by Mr. *Wright* in his errors of *Navigation*, as also by *Leyburn* in his ninth Geometrical Exercise. The Projection of the *Mercators Chart*, and keeping a reckoning at Sea by that, discovers the true Longitude, Latitude, and Distance most exactly. For the Meridians being great Circles of the Globe are alwaies equal, and the Degrees of *Latitude* measured thereupon are also equal:

But the Parallels (to the Equinoctial) are all lesser Circles of the Globe, and towards either Pole alwaies grow lesser and lesser, (as may most evidently appear by the Globe it self if you please to cast your eye upon it:) And therefore the Degrees of Longitude which are measured upon these Parallels do differ in every Latitude: Therefore one side of *Mercators Chart* is the Meridian Line of unequal Parts, and the other side is a Line or Scale of equal Parts only: Upon the Meridian Line of the *Chart* is measured the Difference of Latitudes, and the Distance upon the Rumb. The Difference of Longitude is measured upon the Equinoctial Line, or Line of equal Parts: And in *Trigonometry*, because the sides of all *Triangles* ought to be brought to the same proportional parts one with another, that is the meaning of Meridional

nal Parts ; for inſtead of the unequal Part of the Meridian Line, you take the equal parts of the Line of Lines anſwering thereunto : Now how to find theſe Meridional Parts in reference to the Latitude you have plainly ſet down *pag*. 283. The ſame Rules will alſo ſerve for the Diſtance between any two places, provided that taking the ſame Diſtance in your Compaſſes, you ſet one Foot of your Compaſſes as much below the leſſer Latitude, as the other above the greater Latitude.

Then by the former Rules, *pag*. 283, find the Meridional Parts anſwering to the Degrees on the Meridian Line intercepted betwixt the feet of the Compaſſes, and thoſe are the Meridional Parts anſwering to the Diſtance upon the Rhumb, and to be uſed inſtead thereof.

So that you ſee in ſhort, the difference betwixt Sailing by the *Plain Chart*, and *Mercators Chart*, which differ in this, and this only: That in the Plain Chart both the difference of Longitude and Latitude, as alſo the diſtance are all meaſured by the ſame Scale of equal parts ; but in *Mercators Chart* the difference of Longitude only is meaſured upon the ſcale of equal parts, and the difference of Latitude and diſtance muſt be alwaies meaſured upon, and taken out of the Meridian Line of the Chart: And when either the difference

of

of Latitude or the distance is opposed in Proportion to the Longitude, instead of the Degrees of the Meridian Line you must use the Meridional Parts answering thereunto, which by most are called the Meridional Distance, or Meridional difference of Latitude; but by Mr. *Leyburn* in his Geometrical Exercises, *pag.* 18. They are called the Proper distance, and the proper difference, which I shall retain in some of these following Problems taken out of the same Author.

SECT. 7.

In Gunters Sector *the Meridional Line being placed on the same side with the Line of Lines, you may thereby easily find these Meridional parts by these Rules.*

1. If you would know the difference of Latitude in Meridional parts betwixt two places, the one lying under the Equinoctial, (as the River of *Amazones*) the other in 50 degrees of North Latitude, suppose (the Lizard). Look for 50 on the Meridian Line, and right against it on the Line of Lines you find 57 *d.* 54 *m.* or in decimal parts of a degree, 57 *d.* $\frac{90}{100}$ and these are the Meridional parts required.

2. If both the places have Northerly or Southerly Latitude, as suppose one in 30 *d.* the other in 50 *d.* Extend your Compasses from

from one of the Latitudes to the other, as from 30 *d.* to 50 on the Meridian-Line. The ſame Extent will reach from the beginning of the Line of Lines to 26 *d.* 26 *m.* which is the Meridional parts required.

3. When one of the places hath South-Latitude, and the other North, as ſuppoſe 10 *d.* South, and the other 30 North. The Extent from the beginning of the Meridian Line to 10 degrees ſhall reach from 30 *d.* on the Meridian Line, to 41 *d.* 31 *m.* on the Line of Lines or equal parts, which are the Meridional Parts required.

Knowing then how to find theſe Meridional parts, Note that the difference of Latitude in theſe is called the Meridional difference of Latitude, and the true difference of Latitude is that which is found by ſubſtracting the leſſer Latitude out of the greater, the Remainder is the true Latitude. This being premiſed, I ſhall now proceed to theſe ſeveral Propoſitions.

PROBL. 1.

The difference of Latitude, and difference of Longitude being given or known, to find the Rumb.

As the difference in Latitude is to the diffe-

difference in Longitude, So is the Tangent of 45 to the Tangent of the Rumb.

Suppose the two places be the Lyzard in 50 degrees North, the other St. *Christophers* in 15 *d.* 30 *m.* the proper difference of Latitude is $42\frac{12}{100}$, and the difference of Longitude 68 *d.* 50 *m.* Therefore the Extent on the Line of Numbers from $42\frac{12}{100}$ to 68 *d*, 50 *m.* shall reach the same way from the Tangent of 45, to the Tangent of 58 *d.* 26 *m.* which is the Rumb leading betwixt those 2 places.

PROBL. 2.

The Difference of Latitude and the Rumb given, to find the difference of Longitude.

As Radius, or the Tangent of 45 to the Tangent of the Rumb, so is the proper difference of the Latitude to the difference of Longitude.

PROBL. 3.

By one Latitude, the difference of Longitude and the Rumb given, To find the other Latitude.

As the Radius is to the Co-Tangent of the Rumb from the Meridian, so is the diffe-

difference of Longitude to the proper difference of Latitude.

PROBL. 4.

Having the Latitude of one place (50 *d.*) and the Rumb (33 *d.* 45 *m.*) leading from that place to another unknown; and the diſtance upon the Rumb from the firſt place to the ſecond, (6 *d.*) To find the difference of Longitude ($5\frac{1}{2}$ *d.*)

As the Radius is to the ſine of the Rumb from the Meridian, ſo is the proper diſtance upon the Rumb to the difference of Longitude.

PROBL. 5.

The difference of Longitude, one Latitude and the Rumb given, to find the diſtance.

As the Sine of the Rumb is to the difference of Longitude, ſo is the Radius to the proper diſtance.

PROBL. 6.

Difference of Longitude, and diſtance with the Latitude of one of the places given, to find the Rhumb that leads from one to the other.

As

As the proper distance is to the difference of Longitude, so is Radius to the Sine of the Rumb.

PROBL. 7.

The Longitude and Latitude of two places being given, to find the distance and Rumb.

As the proper difference of Latitude is to the Radius, so is the difference of Longitude to the Tangent of the Rumb from the Meridian: And as the Sine of the Rumb is to the difference of Longitude, so is the Radius to the proper distance.

PROBL. 8.

The difference of Latitudes and their distance upon the Rhumb given, to find the difference of Longitude.

As the proper distance is to Radius, so is the proper difference of Latitudes to the Co-sine of the Rumb from the Meridian: And so is the Sine of the Rhumb from the Meridian to the difference of *Longitude.*

PROBL. 9.

The Difference of *Longitude* of two places, their Distance upon the Rumb, and the

Latitude

Latitude of one of the places being given. To find the Difference of *Latitudes.*

As the proper diſtance is to the Radius, ſo is the difference of *Longitude* to the inclination of the Rumb to the Meridian: And ſo is Co-ſine of the Rhumb to the difference of *Latitudes.*

PROBL. 10.

By one *Latitude*, Diſtance and Rumb, to find the other *Latitude.*

As Sine 90, or Radius is to the Coſine of the Rumb, ſo is the diſtance to the true difference of *Latitude*:

I ſhall now proceed to ſhew you Mr *Philips*'s way of drawing a *Mercators Chart*, and how to meaſure both *Longitude*, *Latitude*, and *Diſtance* thereupon, and to keep an account of a Voyage thereby, as is performed by him in his Geometrical Seaman, which I recommend as a Book of very excellent uſe to Seamen and ſuch as deſire to be more at large informed in all the particulars concerning *Navigation* in all the three kinds: Of Sailing by the *Plain Chart*, *Mercators Chart*, or by the *Arch* of a great *Circle.* I ſhall in this place content my ſelf only to Epitomize the Uſe and Figure of his *Mercators Chart* as follows.

Know-

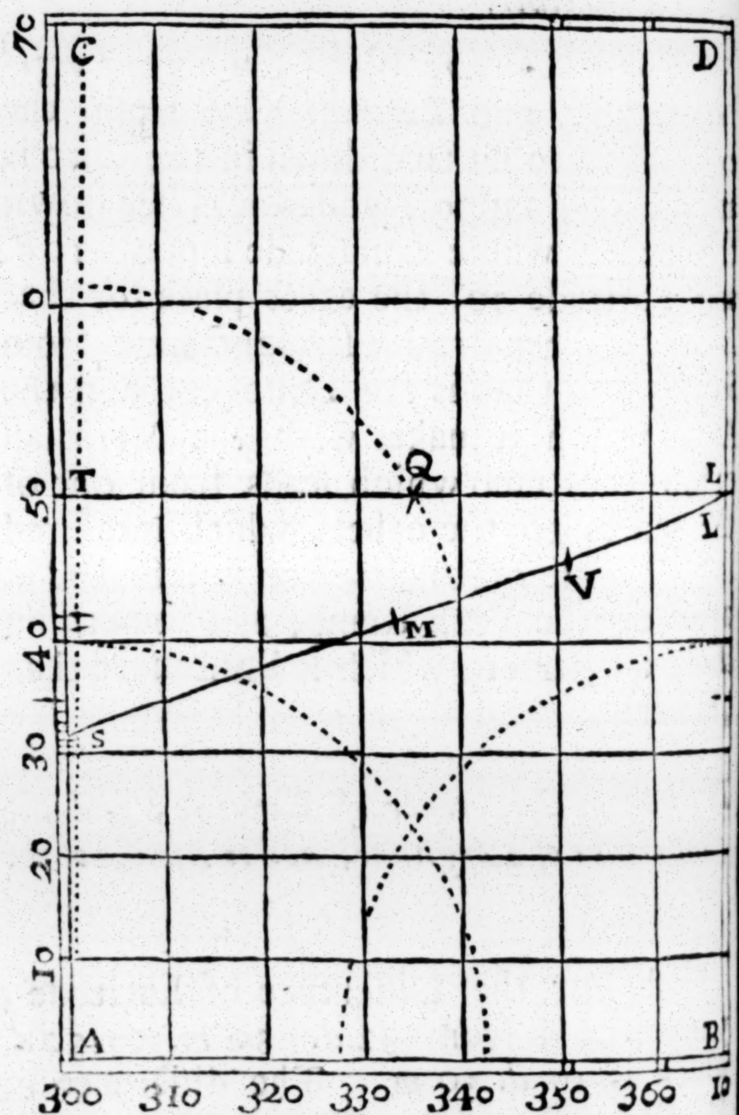
Knowing the Longitude and Latitude of two places, to set them upon the Chart, and to find the Rhumb leading from one of the places to the other.

Suppose *Summers* Island, whose *Longitude* is 300, and *Latitude* 32 *d.* 25 *m.* the exact place to set that down in the Cart is at *S.* Then suppose you would set down the *Lyzard*, whose Longitude suppose 10 *d.* and Latitude 50, the exact place for that is at L. Then draw a streight Line from S to L, and that is the Distance; and the Angle which it makes with the Meridian of S, is the Rumb which leads from one of the places to the other, which measured in your Scale of Rumbs, you will find 71 *d.* 21 *m.* or the sixth Rumb, and something above a Quarter, which is ENE more than ¼ Easterly.

To find the Difference of Latitude, and the difference of Longitude, and to Measure the Distance.

First, For the Difference of Latitude, you see it is from 32 *d.* 30 *m.* to 50 *d.* which is 17 *d.* 30 *m.* The difference of Longitude is 70 *d.* To measure the distance upon the Rumb betwixt S and L, do thus, First, Divide the distance that is in

the Meridian Line between the two Latitudes into two equal parts, which in this Example will fall at 42 Degrees. Then



The Equinoctial Line.

take with your Compasses half the length of

of the Line LS which is LM, and with that distance set one foot of your Compasses in 42, the middle point betwixt the two Latitudes, you will find the other foot will reach downwards to 8 *d.* 30 *m.* and upwards to 63 *d.* 30 *m.* The Degrees in the Meridian Line betwixt these 2 points are 55 degrees, which is 1100 Leagues, and this is the true distance upon the Rhumb betwixt S and L.

To Measure a Parallel Distance between two places.

Suppose the places to be L and T both in the Latitude of 50 degrees, their difference of Longitude being 70, divide the distance betwixt L and T into 2 equal parts which is at Q. Take L Q in your Compasses, and setting one foot in 50 Degrees of the Meridian Line, or the degrees of Latitude, the other foot will reach downwards to 22 *d.* 30 *m.* and upwards to 67 *d.* 30 *m.* and if you count the degrees intercepted between these two points, you will find them 45 *d.* which multiplyed by 20, gives the distance between these two places 900 Leagues.

If two places differ only in Latitude and lye under the same Meridian: The Degrees between them upon the Meridian Line gives the Degrees, which multiplyed

by 2c, bring them into Leagues, or by 60 into *English* Miles.

Having Sailed from the *Lyzard* about the Distance of 15 degrees upon the same Rumb as in the Chart, you would know what Longitude and Latitude you are come to, or the point where the ship then is.

Take 15 degrees in your Compasses from 50 (or L.) which signifies the Lyzard) out of the Meridian line; or the degrees of latitude whatsoever they be, which will reach to 35 *d.* in this Example.

Set this distance upon the Rumb line you Sail upon, which will be at V; lay your Ruler by that mark from 46 and 46 in the Meridian-line, and that is your latitude; do the like for your longitude, and you will find it 351, or thereabouts.

To avoid Sailing in a Parallel that is full East and West at any time.

It will be your best way to steere no nearer to the East or West than the 7 Rumb from the Meridian, for so you may by your observation of the latitude correct your account, which otherwise you cannot do, and in doing thus, you will not go much out of your way; for if the two places be distant 10 degrees, if you sail one half of your way upon the seventh Rumb, you will raise the Pole

Pole very near 1 degree, *viz.* $\frac{554}{1000}$, and if you lay the Pole as much upon the ſeventh Rumb the other way, ſo you will come to the place deſired, having run the ſame diſtance as before, *viz.* 5 leagues. Thus in a Voyage of 10 degrees, or 200 leagues, you go not fully 4 leagues out of your way, which is not one in 50, which will be well recompenſed, becauſe in Sailing thus upon the ſeventh Rumb, you may by the obſervation of the latitude correct your Account.

Note that a Travis may be drawn upon this Chart, as that in *pag.* 277 is done upon the Plain Chart, *mutatis mutandis* in the latitude and Diſtance.

And thus much ſhall ſuffice to have ſpoken of Sailing by the P*lain Chart*, or *Mercators Chart*. Thoſe that have a mind to underſtand the third kind of Sailing by the Arch of a great Circle, they may conſult P*hilips* his Geometrical Seaman and other Authors: I ſhall not here inſiſt upon it, conceiving it to be rather a piece of Nicetie, than any great matter of ſolidity which I diſcern in it; yet withal do take it for an unqueſtionable verity, that it is the neareſt way by reaſon of the Sphærical Form of the Earth and Sea, and circularity of the Terreſtrial Globe. But to let this paſs, I ſhall conclude what I intend to ſay concerning *Na-*

vigation in these following particulars.

1. To shew how many leagues do answer to one degree of longitude in every several latitude.

2. To shew how many leagues do answer to a degree of latitude in every several Rumb.

3. To know the Burden of a Ship, and how to Build a Ship to any proportion or Bigness.

1. To know how many leagues do answer to a degree of longitude in every several latitude.

As the Radius (or sine of 90 *d.*) is to 20 in the line of Numbers, (or 60 if you desire Miles) so is the Co-sine of latitude (at *London* 38 *d.* 38.) to the number of leagues or Miles in that latitude (*viz.* at *London*) 37 $\frac{1}{2}$ Miles, or 12 $\frac{1}{2}$ leagues, *fere.*

To do this by the Sector,

Take 20 leagues out of the line of lines, and make it a Parallel Radius by fitting it over in the sines of 90 and 90, so his Parallel sine taken out of the complement of the latitude, (*viz.* 25. 15.) will give the leagues (*viz.* 18.)

To do this another way:

Suppose you have a Quadrant, the one side whereof being the full Radius may be divided into 20 equal parts, the other side into 60;

60; then with your Compasses take the nearest distance from the complement of the latitude to the side of the Quadrant, and that distance measured upon the line divided into 20 from the Center, will give the leagues upon the line of 60, which will give the miles answerable to one Degree of longitude in that latitude. Other waies there may be for doing this, but these are sufficient.

2. To know how many leagues do answer to a degree of latitude in every several Rumb, or more Plainly to know how many leagues you are to sail upon each Rhumb before you raise or depress the Pole one Degree.

As the complement of the Rhumb is to 20 leagues the measure of one degree at the Meridian, so is the Radius to the leagues answering to one degree upon the Rumb.

As suppose Sailing NEBN from 50 *d.* of latitude, it were required how many leagues the Ship should run before it should come to 51 *d.* of latitude: Because this is the third Rumb, and the inclination thereof 33 *d.* 45 *m.* The Complement thereof is 56 *d.* 15 *m.* Therefore the Extent of the Compasses from 56 *d.* 15 *m.* upon the line of Sines to 20 upon the line of Numbers, the same extent the same way will reach from Radius or the Sine of 90 *d.* to 24, so many leagues must be sailed to raise the Pole 1 *d.*

To do this by the Sector:

Take 20 leagues from the line of lines, and make it a parallel sine of 56 *d.* 15 *m.* and his parallel Radius between 90 and 90 measured in the same line of lines, will shew you 24 for the Number of leagues required.

Sailing with the Ship	North or South	You keep the same Meridian			And 20 *English* Leagues.	Do either Raise or Depress the Pole one Degree.
	Upon the 1st. Rumb	Having changed your Parallel a degree of Latitude, your departure from the Meridian is 4 Leagues.			& 20 1/3	
	Upon the 2	Rhumb.	8 1/3	Leagues.	& 21 1/3	
	3		13 1/3		& 24	
	4		20		& 28 1/3	
	5		30		& 36	
	6		48 1/3		& 52	
	7		101		& 102 2/3	
	East or West	You are still in a Parallel, and neither raise, nor lay the Pole at all.				

To Convert Degrees and Minutes into Hours and Minutes.

Note that 15 Degrees makes an Hour, therefore divide the degrees by 15, the Quotient will be Hours: The Fraction (if any

any remain) multiply by 4, (becauſe every Degree is 4 Minutes of time) and to the Product add every 15 Minute of a Degree (if any be) for ſo many makes a Minute in time. This will exactly reduce Degrees and Minutes to Hours and Minutes. As for Example 118 *d.* by 15. the Quotient is 7 Fraction 13, which multiplyed by 4, is 52. *Total* 7 H. 52 M. as above.

But for your ready finding of your true Northing and Southing, or Eaſting and Weſting, or in other Terms your Difference of Latitude and difference of Longitude by the Rumb and diſtance given, or by knowing how many Leagues you have ſailed upon any Rumb: You have excellent Tables in *Philips*'s Geometrical Sea-man, and in *Sturmy*, or to ſupply thoſe you may have a Traverſe Scale, which is a Sliding Scale, on one ſide having a Line of Numbers, on the other ſide the Rumbs: The uſe of it is thus; Set your diſtance run againſt the Radius (which is the eighth Rumb) and the Number of Leagues againſt the Rumb you ſailed upon, is the difference of Longitude, or your departure from the Meridian, or your Eaſting or Weſting (call it which you will) and the Number of Leagues oppoſite to the Complement of the Rumb is the difference of Latitude, or your Northing or Southing. As for Exam-

ple, ſuppoſe you Sail 57 Leagues upon the third Rumb, and would know how many Leagues you have altered your longitude or latitude: Set 57 the leagues ſailed to Radius (or the eighth Rumb;) look againſt the third Rumb, there you find $31\frac{7}{10}$ leagues for your departure from the Meridian or difference of Longitude, and againſt the complement of the ſaid Rumb, which is 5, or the fifth Rumb, there you will find $47\frac{4}{10}$ leagues, and ſo many leagues is your difference of Latitude: The like of all other.

The Uſe of the Sea Quadrant.

This Inſtrument conſiſts of 3 Vanes and 2 Arches, one of 60, and the other 30, which together make 90 Degrees, and therefore call'd a Quadrant: The 3 Vanes are the ſight Vane upon the 30 Arch, the Shadow-Vane upon the 60 Arch, and the Horizon-Vane at the Angle A: So turn your back of the Sun, and move the ſights, till the ſhadow and the Horizon be ſeen together, and then upon the 60 or leſſer Arch obſerve the Degrees cut by the upper edge of your Shadow-Vane, and add thoſe to the degrees cut by the inſide of the Sight-Vane upon the 30 Arch, which Degrees added together, the ſumm is the Complement of the Altitude, or the Suns diſtance from the Zenith

To

To keep a reckoning upon a Plat of a whole Voyage.

Suppose you Sail from the *Lizard* South-Eastward (which we suppose in the Latitude 50, 00 degrees) and the difference of latitude 3 degrees, which deduct from 50 *d.* and then the latitude the ship is now in is 47 *d*, and suppose your course run 50 leagues Eastward.

Therefore to set the place of your ship upon your Plat, you must use two pair of Compasses, with one pair take the extent between the latitude 50 degrees and 47 degrees, and with that extent set one point of your Compasses in the *Lizard*, the other extend towards the place where the Ship is now, but so that your Compasses stand parallel to a North and South Line. This done, keep one foot of your Compasses in that point, and with the other pair take the departure 50 leagues from the Scale of leagues, then interchange your Compasses, placing this last pair in the point where the other pair stood, set this departure 50 leagues to the Eastward; but so that your Compasses may stand parallel, and to an East and West Line. (This you may find by trying with your other Compasses whether the legs be æquidistant from the next East and West line) This

This ſecond point thus found upon the Plat is the place of the Ship (according to your reckoning) which was required. This I have ſeen practiſed by ſeveral Ship-Maſters upon Maps, and is a very ready way, marking the place where the Ship is with Chalk, which may be rubbed out again when you come at the next Station.

How to know what Burthen a Ship is of.

The general receiv'd Opinion is to take the length at the Keel, the Breadth at the Beam, and his Depth in Hold; which multiply one in the other, and divide the laſt product by 100, gives you his Tunnage, which is the Kings allowance: But Merchants divide by 110; but you muſt note this is of ſuch Ships as are built Taper-wiſe from the Keele, but others think 95 a number reaſonable.

Length 75 foot	}	Product is 28275, which divided by 100, gives $282\frac{3}{4}$ Tunns.
Breadth 29		
Depth 13		

Suppoſing a Ship of 100 *Tun, and you would make another Ship of double Burthen.*

Suppoſe this Ship of 100 Tun be 44 foot long by the Keel, 20 foot broad midſhip Beam,

Beame, 9 foot deep in Hold. Multiply each of theſe Numbers cubically; double the Product, and then extract the Cube Root. As for Example, Multiply 44 by 44, makes 1936, and that again by 44 makes 85184 for the Cube number, which being doubled (becauſe the Ship is to be double the burthen of the other) ſo it makes 170368: Extract the Cubique Root of this Number, and it will yield 55 feet $\frac{437}{1000}$ parts, which is 5 inches, and almoſt $\frac{1}{4}$ inch, and ſo you muſt do by all the other dimenſions of the Ship to find them, and ſo make them proportionable.

What due Proportion ought to be uſed in the Building of all Ships whatſoever.

The due proportion of a Ship is, That the longitude of the Veſſel whatſoever it be more or leſs ought to be divided into 300 equal parts; of the which parts 30 muſt be aſſigned to the Depth, and the Breadth ſhall contain 50, or the $\frac{1}{6}$ part of the longitude, ſo ſhall the Ship be both proportionable, and more ſafe for Traffick.

To conclude pleaſantly, I ſhall here give you Biſhop *Wilkins* device in his *Mathematical Magick* of making a Ship to Sail under Water: The Difficulties herein he reſolves thus.

1. No

1. No land Creatures (eſpecially men) can live without Air wherein they may breathe, and therefore ſuch a Veſſel muſt be very large and ſpacious; ſo as to allow Refrigeration.

2. If ſo, How is it poſſible that they ſhould ſwim or ſail along. To this he ſaies, That thoſe Bodies which are carried on the Water, be they never ſo big or ponderous, (ſuppoſe equal to a City or whole Iſland) yet they will alwaies ſwim on the top, if they be but any thing lighter than ſo much Water as is equal to them in bigneſs.

3. How to poiſe this Veſſel ſo equally, That it ſhall neither riſe too high, nor ſink too low: This he propounds to be done by hanging more or leſs great weights at the bottom.

4. How the motion of this Veſſel muſt be made, which muſt be by Oars put thorough the ſides of the Ships with leather-bags made faſt to the Hole, and tyed cloſe to the Oars, by which means the Oars may move, and yet let in no Water at all.

5. The ſame device doth remedie the 5 and laſt, and greateſt difficulty, which is to take any thing into the Veſſel, or put any thing out of the Veſſel without admiſſion of any Water: How this may be done, he propounds thus; Suppoſe a leather-bag of what bigneſs you pleaſe made faſt

and

and close to a Window round about, being also ty'd close about towards the Window, then any thing that is to be sent out may be safely put into that end within the Ship, which being again close shut or tyed, and the other end loosed, the thing may be safely sent out: And by the like means how to receive any thing into the Ship or Vessel is easie to conceive.

But I shall sail no further in this Discourse, the Voyage in such a Vessel being near as difficult and dangerous to perform, as to fly into his *New World of the Moon*, though he tells you *pag.* 219. That if a Body were above the Sphere of the Magnetical Vertue of the Earth (which he supposes 20 Miles upwards, or so) he might there stand as firmly in the open Air, as he can now upon the Ground.

Or Secondly, If this Sailing under Water could be performed, it would not be of much usefulness; therefore shall rank this amongst those other conceits which are demonstrable, but not practicable, such as pulling up Trees by the Roots with a single hair, or by the breath of a mans mouth: Or *Pythagoras* writing in the Moon by reflection: Or *Archimedes* his moving *datum pondus cum data potentia*; that if he did but know where to stand and fasten his Instrument, he could move the whole Terrestrial Globe.

Globe. More uſeful (if at all practicable) is his perpetual motion; but both uſeful and practicable, and in many places practic'd and us'd is his Sailing Chariot, like that in *Holland*, which with the Wind would paſs from *Scheveling* to *Putten* in two hours, being diſtant more than 42 *Engliſh* Miles.

But paying due Honour and reſpect to the mention of this Reverend, Learned, and Ingenious Author, I ſhall here conclude what I have to ſay of *Geometry* in general, and in particular as to *Surveying* and *Navigation*; and proceed according to my propounded Method to more ſolid and ſerious *Mathematical Propoſitions* and *Practical Concluſions in Aſtronomy.*

THE

The Third BOOK.

OF ASTRONOMY.

CHAP. I.

HAving hitherto treated of *Arithmetick* in the four kinds of it, *Common*, *Decimal*, *Logarithmetical*, and *Instrumental*, and alſo of *Geometry* both general in the Reſolution of Geometrical Problems and Propoſitions; and in the producing, reducing, and meaſuring the three principal Geometrical Figures: And likewiſe of *Geometry* in ſpecial applyed to *Surveying* and *Navigation*: I come now in the third place to treat of the third *Mathematical Liberal Science*, which is *Aſtronomy*: And as *Geometry* may be ſaid to meaſure the Earth, or this whole Terreſtrial Globe of Sea and Land, by taking Heights and Diſtances

ſtances at Land in *Surveying*, and by taking Heights and Diſtances at Sea in *Navigation*: So *Aſtronomy* may be ſaid to meaſure the Heavens by taking the Heights and Diſtances of thoſe Heavenly Bodies, *viz.* The Planets and Fixed Stars.

But that theſe Heights and Diſtances here principally meant and intended, may be entirely handled, and that other things which concern *Aſtronomy*, may be the better underſtood, I ſhall premiſe theſe ſeveral particulars, and proceed in this Method.

1. I ſhall give you the number and names of theſe Heavenly Bodies, *viz.* the Planets and Fixed Stars.

2. I ſhall ſhew you their Order, both in the Ptolemaick and Copernican *Syſtems* of the World, and give you the Reaſons given on both ſides, *pro & con* for each Syſteme.

3. I ſhall ſhew you their Periodical Revolutions.

4 I ſhall ſhew you the Magnitudes.

5. Their Heights or Diſtances from the Earth:

Which will lead me by the hand as it were in the laſt place to ſpeak of theſe other Heights and Diſtances which I make the Sum and Subſtance of *Aſtronomy* as it ſtands in Analogy with *Geometry* in this place: And theſe, to wit, the latter ſort of Heights and

and Distances are properly Questions of the Sphear, therefore I shall explain to you the *Projection of the Sphere in plano*, and the *Resolution of Spherical Triangles* grounded thereupon; by which *Spherical Triangles* these *Astronomical* Questions are resolv'd; as *Surveying* and *Navigation* are by *Plain Triangles*. But to proceed according to my proposed Method.

CHAP. II.

To give you the Number and Names of the Planets and Fixed Stars.

1. AS for the Planets, all agree as to the Number that there are 7, but which these seven are there is some difference; *Aristotle*, *Ptolomy*, and the rest of the *Peripateticks* do number them thus, *Saturn*, *Jupiter*, *Mars*, *Sol*, *Venus*, *Mercury*, *Luna*; making the Earth the Center of the World: But *Pythagoras*, *Copernicus*, and the Ancient *Pythagoreans* do make the Sun the Center of the World, and this Earth to be a Planet instead of the Sun, and to move in the same Orb which is ascribed to the Sun by *Ptolomy*, and so numbers the Planets thus, *Saturn*, *Jupiter*, *Mars*, *Tellus*, *Luna*, *Venus*, *Mercury*, and *Sol* the Center, and

and ſome will not have the Moon to be a primary Planet, but only a ſecondary, and that all the reſt of the Planets as well as this earth have their Moons to move about them, and for *Jupiter* there are 4 obſerved to move about him, which they call *Satellites*; and *Saturn* alſo is obſerved to have his concomitants.

Secondly, For the number of the Fixed Stars, they are vulgarly and commonly conceived to be innumerable, yet *Mathematicians* do ſay they may be numbred, and do take notice only of 1025, or at moſt of 1161, comprized in 48 Conſtellations,

	21	In the Northern Hemiſphere.
	15	In the Southern.
	12	In the Zodiack, or the Zodietical Conſtellations, being no other than the 12 Signs.
In all	48	

The

The Names of the *Northern* Conſtellations are

	Containing Stars
1 Urſa Minor	7
That in the end of the Tail is called the Pole Star.	
2 Urſa Major.	27
3 Draco.	31
4 Cephus.	11
5 Bootes.	22
6 Corona Borea	8
7 Engonaſis, *or* Hercules	29
8 Lyra.	10
9 Olor, *or* Cignus.	17
10 Caſſiopeia	13
11 Perſeus	26
12 Auriga	14
12 Conſtellations	215 Stars.

	Containing Stars
13 Ophiucus, *or* Serpentarius.	24
14 *S*erpens	18
15 Sagitra, *or* the Arrow	5
16 Aquila, *or* Vultur volans	9
17 Delphinus the Dolphin	13
18 Equiculus, *or* little Horſe.	4
19 Pegaſus, *or* the winged Horſe.	20
20 Andromeda	23
21 Triangulum the Triangle	4
9 Conſtellations	Stars 117
12	215
21 Northern	332

The

The Names of the 15 Southern Constellations.

N°.			N°.		
1	Cetus the Whale	22	9	Crater the Cup	
2	Fluvius Eridanus	34	10	Corvus the Crow	
3	Lepus the Hare	12	11	Centaurus	
4	Orion	38	12	Lupus the Wolf	
5	Canis Major	18	13	Ara the Altar	
6	Canis Minor	2	14	Corona Austrina the South Garland.	
7	Argo Navis	41	15	Piscis Austrinus	
8	Hydra	25			
8	Constellations. Stars	192	7	Constellations. Stars	1
			8		19
			15	South Constellat.	29

The Names of the 12 Zodiatical Constellations,

N°.			N°.		
1	Aries the Ram	13	7	Libra	
2	Taurus	23	8	Scorpio	2
3	Gemini	18	9	Sagittarius	3
4	Cancer	9	10	Capricornus	2
5	Leo	27	11	Aquarius	4
6	Virgo	26	12	Piscis	2
6	Constellations. Stars	116	6	Constellations. Stars	1
			6		11
			12	Constellations.	27

The

These are the 48 Constellations taken out of Mr. *Moxons* Book *De Globis*, where you may find their several Poetical stories, which are not unpleasant to read: And thus much shall serve for the Number and Names both of the Planets and Fixed Stars.

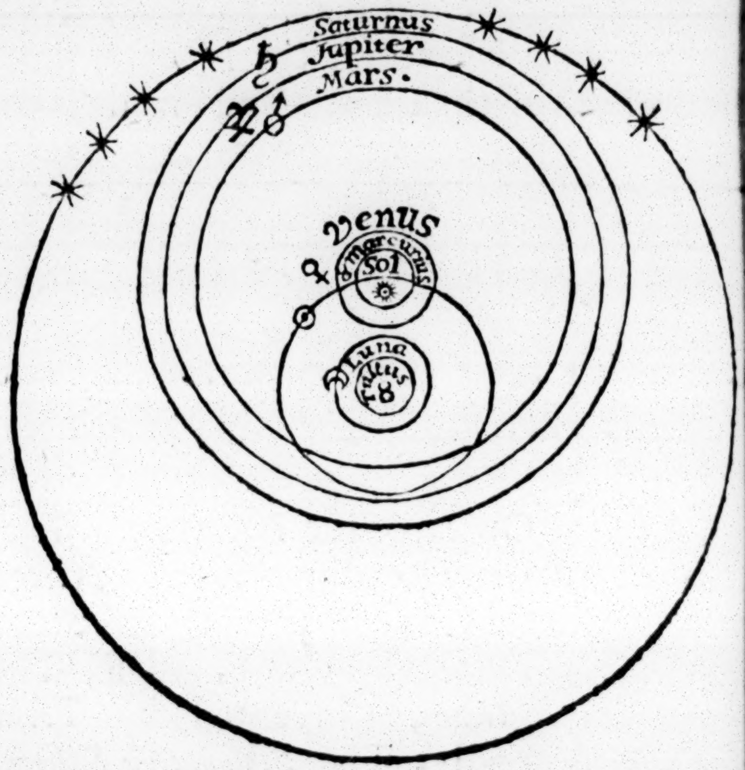
CHAP. III.

Concerning their Order in the Systeme of the World.

AS to the Order of the Planets and Fixed Stars, my Reading affords me three several Systemes (though possibly there may be more:) These three are, First, The *Tychonican Systeme*, (invented by *Ticho Brahe*.) Secondly, The *Ptolomaick Systeme* (invented and maintained by *Ptolomy* and the *Peripateticks*.) Thirdly, The *Copernican Systeme*, being the Ancient Opinion of *Pythagoras* and his followers, but revived by *Copernicus*, and since imbraced by most Modern Mathematicians, though Vulgar Capacities do take it for a strange thing, *monstrum Horrendum*, &c. I shall give you these three several *Systems* in order, and first, The *Tychonican*.

This

The *Tychonican Systeme.*

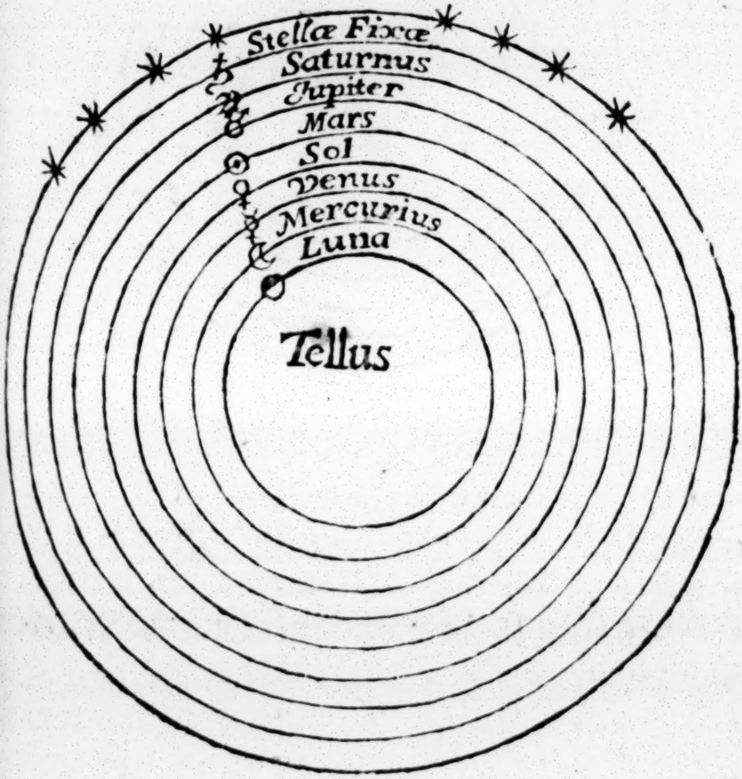


This Systeme was Invented by the Illustrious *Ticho Brahe* a Nobleman of *Denmark*, Lord of *Knudthorp* in the Island of *Shonen*, not far from *Elsenbourgh*, who framed this Hypothesis as a mean betwixt *Ptolomy* and *Copernicus*.

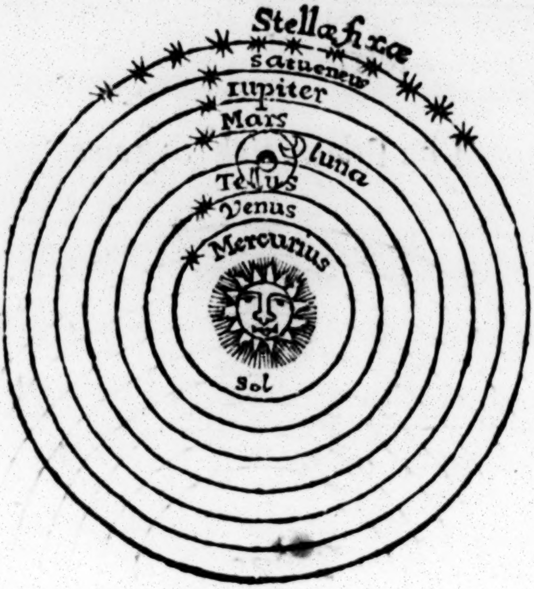
By this he supposeth that *Mercury*, *Venus*, and all the other Planets (except the Moon) in their motion, respect the Sun as their Center: So that *Saturn* in opposition to the Sun

Sun is nearer the Earth than *Venus* in Apogæon, and that *Mars* in oppoſition to the Sun, is nearer to the Earth than the Sun it ſelf, as may appear by Inſpection from the Hypotheſis it ſelf.

The *Ptolomaick Syſteme.*



The *Copernican Systeme.*



For these two Systemes, *viz.* the *Ptolomaick* and the *Copernican*, I shall give the reasons *pro & con* out of *Varenius*, and because they cannot be better expressed, I shall give you them in his own words, as follows, *Geographia Generalis* abbreviated on this manner, *pag.* 34, 35, 36, 37, 39, and 40.

Rationes Copernicani *Systematis.*

1. Quoniam tantus est numerus stellarum quæ horis 24 circa Tellurem circuitum videntur perficere, & hæc apparentia per unius

unius Telluris (manentis in ſuo loco) motum explicari poteſt, ideo magis rationi conſentaneum eſt hunc ſtatuere quam illum; ſicut nobis in Navi ſedentibus, & appropinquantibus ad ſtationem multarum Navium quæ apparent nobis appropinquare, non ideo hiſce motum aſcribimus; & omnino cum natura non ſoleat facere per plura quod per pauca poteſt, veriſimile eſt, in hac quoq; re illud obſervatum eſſe.

2. Quia incredibilis & omnem cogitationem ſuperans celeritas motus ſtellarum illius foret: etenim cùm infinito ferè ſpatio a Tellure abſint, & vaſtiſſimus orbis illis percurrendus ſit, uno horæ minuto ad minimum per Centena millia milliarium ferri deberent. Contra, Si Telluri motus aſcribatur hic primus, manet hæc in ſuo loco, neq; de minima celeritate timendum eſt, quia Tellus circa ſuum axem rotatur.

3. Accedit argumento huic majus Robur ſi comparemus vaſtitatem corporum cœleſtium cum terreſtri. Etenim cum Sol ad minimum ducenties major ſit Tellure, ſtellæ vero fixæ vel millies majores, cui non veriſimilius fiat Tellurem rotari circa ſuum Axem motu naturali, quam tanta corpora incredibili pernicitate de loco in locum moveri?

4. Quoniam omnes Aſtronomi illuſtriores cum Tychone coacti apparentiis jam

negant Orbes solidos; quibus antiqui ad faciliorem stellarum motus Hypothesin utebantur. Ideo multo incredibilior videtur illarum circumlatio circa Tellurem. Negant autem Orbes solidos, quia si hi essent, concedenda esset penetratio orbium mutua, cum quidam planetæ in alterius alicujus Sphærâ deprehendantur frequenter.

5. Nulla Ratio reddi potest cur Stellæ circa Tellurem moveantur, cùm contra cur Terra & reliqui planetæ circa solem cum reliquis planetis moveantur, aliqua possit dari Ratio.

6. Quia nec Polus, nec Axis est realis circa quem Stellæ ferri ponuntur; contra in Tellure & polus & Axis est.

7. Quia multo facilior est Navigatio ab occidente in orientem, quam ab oriente in occidentem. Etenim ex *Europa* in *Indiam* navigatur mensibus circiter quatuor, ex *India* in *Europam* reditur sex mensium spatio circiter. Nimirum quia in illa Navigatione in eandem plagam cum Tellure moventur, in hac vero in contrariam.

8. Quia omnes apparentiæ cælestes, ortus, occasus syderum, dierum incrementum, *&c.* possunt explicari, si Tellurem moveri ponamus.

Imprimis autem hujus Hypothesis commoditas & necessitas conspicitur in admirandis illis Planetarum affectionibus ad quas explicandas

candas *Ptolemaici* multos circulos Epicyclos & eccentricos ſine ulla ratione excogitare coguntur: *Copernicani* autem illas ex Telluris motu ſecundo circa ſolem facili negotio deducunt ita, ut manifeſtam illarum cauſam reddant, & adeo facilem ut vel indocti illam capere poſſint, Nimirum.

1. Cur Planetæ interdum retrogradi videantur, & quidem *Saturnus* ſæpiùs & diutiùs quam *Jupiter*, *Jupiter* quam *Mars*, &c. interdum celeriori motu ferri, interdum ſtationarii eſſe.

2. Cur *Venus* & *Mercurius* nunquam poſſint tota nocte conſpicui eſſe.

3. Cur *Venus* nunquam majori a ſole intervallo, quam ſexaginta graduum, *Mercurius* non majori, quam triginta graduum intervallo diſcedat, & ideo nunquam oppoſiti conſpiciantur ſoli.

4. Cur *Venus* ejuſdem diei & veſpere poſt ſolem & mane ante ſolem poſſit conſpici.

Plures apparentias afferre ſuperſedeo: ſed illæ præcipue ſunt quæ per hunc motum Telluris adeo appoſite & jucunde explicentur, ut admirandum potius eſſet ſi Tellus non moveretur talibus apparentibus Phænomenis.

9. Sol non tantum fons lucis eſt, quæ tanquam clariſſima fax illuminat Tellurem, Lunam, Venerem & reliquos ſine dubio planetas, ſed etiam focus caloris & vitalis

ſpiritus quo totum hoc univerſum foveri & ſuſtentari videtur. Ideo medium locum omnium obtinere, & hos circa eum moveri probabile.

10. Sole collocato in medio redditur aliqua cauſa quare reliqui planetæ & Tellus circa eum ferantur, nimirum quia ſol vaſtiſſimum corpus eſt, & magnis viribus præditum, ideo reliquos planetas ad motum excitat.

11. Solem circa Axem ſuum rotari probant obſervationes Galilæi & Scheineri de maculis ſolaribus. Hac igitur ratione reliquis planetis circumeundi cauſa exiſtit, nec videtur ei alius motus attribuendus.

12. Si Telluri locum inter Martem & venerem, ſoli vero centrum attribuamus, apte reſpondet motus ſingulorum planetarum diſtantiæ a Centro, quod in Ptolemaica Hypotheſi non fieri patet ex motuum ſolis, Veneris & Mercurii conſideratione.

Verum enim vero *Ariſtotelici* cum *Ptolemaicis* ſententiam *Pythagoricorum* pluribus impugnat Argumentis & ſuam Hypotheſin probare conantur hiſce rationibus.

Rationes Ptolemaici *Syſtematis ſequuntur.*

1. Terram eſſe ineptam ad motum propter gravitatem; & Gravia feruntur ad Centrum mundi, Tellus autem graviſſimum corpus

corpus eſt, itaq; centrum illud occupat.

2. Gravia a Tellure diſcederent verſus centrum univerſi, niſi in Tellure hoc centrum eſſet.

3. Partes Telluris moveri naturaliter motu recto ad centrum, ergo circularem motum eſſe contra naturam ejus.

4. Si Tellus moveretur, lapidem e turri demiſſum non poſſe cadere ad pedem turris, vel globum è tormento exploſum verſus orientem ad metam aliquam (vel etiam ſi avis quædam ad orientem volaret) non poſſe hanc attingere, ſi meta cum tota tellure verſus orientem moveretur, vel ſaltem celeriorem fore attactum, ſi ad occidentem emiſſus eſſet globus.

5. Neq; turres neq; ædificia conſiſtere poſſe, ſed propter illum Telluris motum collapſura eſſe, neq; homines à vertigine immunes fore.

6. Quia videmus (inquiunt) ſtellas mutare locum, non autem Tellurem.

7. Quia Tellus eſt in centro mundi: centrum autem non movetur.

8. Quia ſacræ literæ ſtabilitatem Telluris confirmant.

Ad hæc Argumenta Copernicani *reſpondere ſolent hunc in modum.*

1. Ad primum negando totam Tellurem

gravem esse; namq; gravitas est partium ad totum Homogeneum tendentia: Et talis gravitas quoq; in lunæ partibus & solis.

2. Et aliorum gravium motus non ad Centrum universi sed ad Corpus Homogeneum est, ut ex partibus Lunæ, Solis, magnetis probatur.

3. Motum illum rectum esse partium Telluris, non totius Telluris, atq; hujus circularem motum non impedire illarum rectilineam Lationem, quod declaratur a partibus Lunæ & Solis.

4. Ad hoc Argumentum respondetur triplici modo 1: Primo enim gravia talia non ad Centrum primario feruntur, sed ad ipsam tellurem, & ideo brevissima linea ad superficiem ejus; brevissima autem est hæc quæ turri respondet: Sicut ferrum non ad centrum magnetis, sed ad magnetem tendit. 2. Totus aer adhæret telluri & cum hac movetur, ideo etiam talia gravia demissa simul hunc circularem motum acquirunt & moventur tanquam in vase. 3. Gassendus crebrâ experientiâ demonstravit, quod si ex moto corpore aliquid projiciatur, hoc Projectum etiam illo motu corporis moveri, exempli gratiâ lapidem dejectum è fastigio mali navis celerrime motæ, tamen non relinqui à nave, sed ad pedem mali decidere: Et è pede mali explosum perpendiculariter è sclopeto globum, rursus perpendiculariter decidere

decidere. Itaque allata objectio nihil valet: *Or if a Man jump in a Ship, he will not be able to pass further, whether he jumps with, or against the motion of the Ship.*

5. Ad quintum Argumentum dicimus tale quid locum non habere: quia motus est æquabilis, nec in aliud corpus impinget, & ædificia tanquam corpora gravia & Telluri homogenea moventur tanquam in navi. Etenim in navi celerrimè etiam vel tardissimè mota, si modo æquabilis sit navigatio hoc est sine fluctibus, & aquâ planâ, deprehendimus erecta corpora non everti, imo pocula vino plena nihil effundere.

6. Ad sextum, dicimus mutationem loci stellarum non sentiri, sed situs respectu nostri mutationem deprehendimus: potest autem hæc situs mutatio animadverti & esse, sive nos cum Tellure sive nobis immotis stellæ moveantur, vel etiam & nos & stellæ.

7. In septima objectione & major & minor falsa est, vel saltem dubia.

8. Ad Octavum respondetur. 1. Sacram scripturam in rebus Physicis Loqui secundum apparentiam & vulgi captum, exempli gratia quando Luna cum Sole dicitur magnum luminare quod ad noctem illuminandum creatum sit, cum tamen nec Luna magna sit respectu stellarum & Telluris, nec proprium lumen habeat, neq; omnibus no-

ctibus illuminat terram. Ita solem ab extremitate ire, & ad extremitatem redire dicit scriptura, cum tamen revera talis extremitas nulla sit. Ita in *Jobi* libro attribuitur Telluri figura plana & quadrata, cui columnæ sint suppositæ, quibus innitatur, quod quidem haudquaquam ita intelligendum esse, vel vulgus novit. Plura loca adduci possint: sed sufficiunt hæc; namq; sacræ literæ nobis non ad philosophandum sed ad pietatem colendam concessæ sunt. Secundò Loca quædam scripturæ adduci solent, quæ non de immobilitate ejus, sed de constantia & duratione loquuntur, ut locus ille quem e *Jobo* attulimus.

Thus you have the Reasons both *pro & con* laid down by *Varenius*, both for the *Ptolemaick* and *Copernican Systemes* in reference to the order wherein these Planets and Fixed Stars are placed: I shall leave it to every one to be of what Opinion he sees the most reason for: But however cannot pass by what I find in that great Mathematician Bishop *Wilkins* after that he hath argued seriously for the *Copernican*; he adds jocantly, That they that would have the Earth stand still, and the Sun to move, are like the Cook who would not rost his Meat by turning the meat about to the Fire, but rather by turning the Fire round about the Meat. (*World in the Moon.*

If

If the Reader desire to know more concerning these matters, let him read the foresaid Author, and *Vincent Wing* his *Harmonicon cœleste*, and such as have treated largely of these matters. These may suffice for my present purpose, and so shall pass on to the Third particular propounded to be considered, which is the Motion or Periodical Revolution of these Heavenly Bodies.

CHAP. IV.

Concerning the Motions and Periodical Revolutions of the Planets and Fixed Stars.

ACcording to the two different Hypothesis of *Ptolomy* and *Copernicus*, there are you may see two different Centers to their several Systemes, *viz.* the Earth and the Sun, the one standing still, the other is supposed to move in its proper Orb betwixt *Venus* and *Mars*: If then according to *Ptolomy* the Earth be the Center, then must the Sun move, and also the Fixed Stars, or the Starry Firmament.

1. The Sun must move about the Earth in 24 hours, making Day and Night, which according

according to its proper Orbes Diſtance from the Earth of 1142 Semidiameters muſt be above 7570 *Engliſh* Miles in a Minute of time, and finiſheth his periodical Revolution thorough each ſide in the Zodiack, call'd his ſecond motion in a year, or 365 days 6 hours, *ferè*, *Wing* 365 days 5 hours 48 Minutes, 55 Seconds, by this making Winter and Summer, Spring and Autumn.

2. The Earth ſuppoſed the Center, the Starry Firmament muſt alſo move round about the Earth in 24 hours, and the motion of the Starry Firmament being proportionable to its diſtance from the Earth, a Star in the Æquator muſt move 12598666 *Engliſh* Miles in an hour, or 209974 in a Minute, ſo that if an Horſeman ſhould ride every day 40 Miles, he could not ride ſuch a Compaſs in a 1000 years as the Starry Firmament moves in an hour, which is more than if one ſhould move about the Earth 1000 times in an hour, and quicker than poſſible can be imagin'd; and if a Star ſhould fly in the Air about the Earth with ſuch a prodigious quickneſs, it would burn and conſume all the World here below: This made *Copernicus* not unadviſedly to attribute this motion to the Earth, and that the Orb of the Fixed Stars ſtands ſtill, as well as the Stars in the ſaid Orb are fixed in their places and due diſtances from

from one another. Suppoſing then with *Copernicus* the Sun to be the Center of the Univerſe; (but allowing it withal a circumrotation upon its own Axis) I ſay, ſuppoſing the Sun the Center, there muſt be aſcribed to the Earth at leaſt a two-fold motion: The firſt is its Diurnal motion, anſwerable to the *Ptolomaick* Diurnal motion of the Sun and Stars: This performed upon its own Axis in 24 hours, and conſtitutes day and night. The ſecond motion of the Earth is his motion *a loco in locum* in its proper Orb betwixt *Venus* and *Mars*, as you may ſee in the Syſteme, and this is performed in a year, or 365 days, 6 hours, *fere*; or 365 *d.* 5 *h.* 48′. 55″. inſtead of the periodical revolution of the Sun according to *Ptolomy*, and this conſtitutes the 4 ſeaſons of the year, Spring, Summer, Autumne, and Winter; as is excellently demonſtrated in Biſhop *Wilkins*'s *World in the Moon.*

Now as to the former of theſe motions, or *ſecundum primum Telluris motum* upon its own Axis; you muſt know that every place upon the Terreſtrial Globe (as you may plainly ſee by turning about an Artificial one) doth not move alike, but places under the Equinoctial do move the faſteſt or quickeſt, and nearer either of the Poles they move ſlower, in ſuch proportion as the

the Degrees in each parallel of Latitude are greater or less: Therefore to know how many Miles in an hour any particular place is carried by this motion, take this proportion or Analogy. As the Radius is to the Co-sine of the Latitude; so is 15 *d.* or 900 *English* Miles to the Number of Degrees or Miles required.

As for Example, *Amsterdam* is in the Latitude of 52 *d.* 23 *m.* its complement, or co-sine is 37 *d.* 37 *m.* so that as 90, or Radius is to 37 *d.* 37 *m.* so is 900 to 548 *English* miles; or so is 225 *German* Miles, (4 *English* miles making one *German*) to 137 *German* Miles; or so is 15 degrees to about 9 degrees, which *Amsterdam* moves in an hour, or 9 of these *German* miles in 4 minutes of time: Now how these Numbers 15, 225, and 900 come to be the third number in this proportion; note that 15 *d.* is an hour in the Æquator, these multiplyed by 15, produceth 225 *German* Miles, and multiplyed by 60, produceth 900 *English* Miles, the Æquator passing so many Miles or Degrees in an hour, and so to run the whole circumference in 24 *hours*, 21600.

Thus much shall suffice to have spoken of the motion of the Earth, or the Sun, and of the Fixed Stars for the rest.

Luna is the loweſt of all the Planets according to *Ptolomy*, or only a ſecondary Planet according to ſome *Copernicans*, finiſhes her courſe in 27 dayes, and almoſt 8 hours.

Mercury finiſhes his Revolution about the Sun in 87 dayes, 23 hours, 15 Minutes, 53 ſeconds according to *Kepler*, and is never diſtant from the Sun above 30 degrees.

Venus performes its courſe in 224 dayes, 16 hours, 49 minutes, 24 ſeconds, and is never diſtant from the Sun above 60 degrees.

Mars finiſhes his courſe in 2 years, or more ſtrictly in 686 days, 23 hours, 27 Minutes, and 30 ſeconds; or according to *Wing* in 1 year, 321 dayes, 23 hours, 32 minutes.

Jupiter moveth through the Zodiack in the ſpace of 11 years, 10 moneths, and almoſt 16 dayes, or according to *Kepler* in 4332 dayes, 12 hours, 20 Minutes, 25 ſeconds, according to *Wing* 11 years, 317 dayes, 14 hours, 49 minutes: His Satellites or Attendants circumvolveth him in time correſponding to their diſtances from him, the firſt and next him in 1 day, 18 hours; the third ſecond in 3 dayes, 13 hours; the in 7 dayes, 4 hours, the fourth and outermoſt in 16 dayes, 5 hours.

Saturn, the higheſt of all the Planets finiſheth his periodical courſe in 29 years, 5 moneths,

moneths, 15 dayes, according to *Alfraganus*, but more exactly according to *Kepler*, in 10759 dayes, 6 hours, 36 minutes, and 26 seconds, or according to *Wing*, *pag.* 63 *Harm.* in 29 years, 174 dayes, 4 hours, 58 minutes. Saturnes Moon, or Concomitant is observed to move about him in 16 dayes, and note that the circumrotation of the Sun upon his own Axis is in 26 dayes, and the Earth in 24 hours, (together with the rest of the Planets that move about the Sun) These all move from *West* to *East*, also *Saturn* and *Jupiters* concomitants about them do keep the same course from West to East.

And besides these, it is not improbable but that every primary planet hath his proper Revolution upon his own Axis (as the Sun and Earth have theirs:) and that they have their Moons or Concomitants moving about them as our Earth hath: Nay, my Author adds that it is not unlikely but that the Sun being of the same matter with the Fixed Stars, they also may all have their Planets or Worlds moved about him, as our Sun hath his. And *Des Cartes* hath shown us (saith Doctor *Power*,) That every Fixed Star is a Sun, and is set in the Center of a *Vortex* or Planetary Systeme as ours is; and that they are as far remote one off another, as ours is off them; and that all our whole Planetary *Vortex* shrinks into nothing

thing, if compared with theſe innumerable Syſtems above us.

No queſtion but all theſe Planets are illuminated by the Sun (as is obſerved by the Teleſcope:) And that the Earth ſhines at a Diſtance with like ſplendor as her fellow Planets, as will eaſily appear by her illuminating the Darker part of the Moons ſubvolvane or lower Hemiſphere, as is commonly ſeen a little before or after the change, for then to the Moon the appearance of the Earths light is near the full: But to make no further Digreſſion. I come in the next place to ſhew the ſeveral magnitudes of theſe Cœleſtial Bodies.

CHAP. V.

Concerning the Magnitude of the Planets and Fixed Stars, and their Diſtances from the Earth.

IT is not my Intention to ſhew you the Theory or way of Calculation, or finding theſe Magnitudes, and Diſtances, or the ſeveral motions of the Planets: Theſe require a large Treatiſe of it ſelf, and is learnedly and largely done by Mr. *Vincent Wing* in that excellent Book of his *Harmonicon Cœleſte*,

Cœleste, to which I refer the Reader: And as I have done their Motions, so shall only *per Transitum* lay down their Magnitudes and Distances as I find them in Authors and so proceed to questions of the Sphear which is principally here intended. For their Magnitudes, take these Observations

1. These Planets have their several Orbs wherein they move, the Semidiameter of which Orbs compared to the Semidiameter of the Earth have this proportion.

			Deg.	Min.
Of what Parts the Semidiameter of the Earth is one, of the same the Semidiameter of the Orb of	Luna	is	48	56
	Mercury		116	3
	Venus		641	45
	Sol		1165	23
	Mars		5032	4
	Jupiter		11611	31
	Saturn		17225	16

2. The Diameter of	Saturn	Compared with the Diameter of the Earth, is as	9 to 2
	Jupiter		32 to 7
	Mars		7 to 6
	Sol		11 to 2
	Venus		3 to 10
	Mercury		1 to 28
	Luna.		5 to 17

The Diameter of the Sun compared with the Diameter of the Moon beareth the same proportion that is betwixt 187 and 10,

The

The Diameter of a Star of the 1, 2, 3, 4, 5, 6 Magnitude is to the Earths Diameter, as 119 to 4, 269 to 60, 25 to 6, 19 to 5, 19 to 36, 21 to 8.

The Diameter of a Star of the	1	Magnitude is to the Earths Diameter, as	119 to 4
	2		269 to 60
	3		25 to 6
	4		19 to 5
	5		19 to 36
	6		21 to 8

3. The Magnitude or Proportion of the Body or Globe of theſe Planets to the Globe or Body of the Earth according to the beſt obſervations of Mr. *Vincent Wing* in his *Harmonicon Cœleſte*, *lib.* 3. *chap.* 25, 26, *fol.* 95, 96. are as follows.

Saturn, Jupiter, Sol is greater than the Earth:
Saturn $9\frac{283}{3176}$ times.
Jupiter $9\frac{932}{22859}$ times.
Sol 343 times.
and greater than the Moon $15765\frac{103}{300}$ times.

Mars, Venus, Mercury, Luna is leſſer than the Earth:
Mars $55\frac{66}{321}$ times.
Venus $26\frac{5077}{12167}$ times.
Mercury $117\frac{371}{2744}$ times.
Luna $45\frac{29}{30}$ times.

There are divers other Authors which do much differ from him in theſe Magnitudes, but I take him to be neareſt the Truth.

The

The Magnitudes of the Fixed Stars, or their Proportions compared with the Globe of the Earth, which take as follows out of *Moxon* de Globis.

A Star of the	Magnitude is to the Globe of the Earth.	Therefore bigger than the Earth.
1	As 6859 to 64	107 [illegible]
2	As 19465109 to 216000	90 [illegible]
3	As 15625 to 216	72 [illegible]
4	As 6850 to 125	54 [illegible]
5	As 1685159 to 46656	36 [illegible]
6	As 9261 to 512	18 [illegible]

Times.

This

This may ſeem incredible to viſual ſenſe, but if the vaſt Diſtance of thoſe Huge Bodies, and the Diminutive quality of Diſtance be conſidered, reaſon will be rectifed: For according to Mr. *John Dee's* computation, the Starry Firmament is diſtant from the Earth 20081' Semidiameters, which according to 60 Miles in a degree is 69006540 Miles: As is noted in its proper place.

And well may the Fixed Stars ſeem little to us; for Doctor *Power* ſaies, That if one ſtood in the Firmament to behold this Earth of ours, it could never be ſeen at all, and therefore if it were annihilated, would never be miſſed: So much doth Diſtance diminiſh the Object.

Now for their Diſtances from this Globe of the Earth, I ſhall give you them from *Varenius* in his own words, *Geog. gen. lib.* 1. *c.* 6.

Si quæras quantum Tellus & nos in Tellure exiſtentes, diſtemus a Planetis, ſciendum eſt, Non eſſe unam & eandem perpetuo diſtantiam ſed ſingulis diebus mutari, & propterea Aſtronomi tres diſtantiarum gradus recenſent minimam, mediam, maximam. Media Diſtantia Telluris a reliquis Planetis juxta plurimos Aſtronomos eſt hæc.

A

A Luna *distat Semidiametris suis*	60	
A Mercurio	110	
A Venere	700	
A Sole	1150	
A Marte	5000	*circiter.*
A Jove	11000	*circiter.*
A Saturno	18000	

Verum enim vero omnino incerta est Martis, Jovis, *&* Saturni *&* S*tellarum fixarum distantia, ob defectum parallaxis. In* Copernicanâ *Hypothesi distantia variatur non tantum a Planetarum motu sed etiam a motu, ipsius Telluris.*

John Seller's Atlas Cœlestis.

1. The *Sun* performs his Revolution upon his own Axis in 26 dayes.

2, *Mercury* performs his course in his Ellipsis in 88 dayes: His proper diurnal motion is 4 *d.* 5'. 12". the Circuit of his Sphere is 12059773 miles, so that he wheels in a day 137040 miles in an hour, 5710 Miles, and in a Minute 91 miles.

3. *Venus* makes her periodical Revolution in her Ellipsis about the Body of the Sun in 224 dayes. It is from the Sun to the Sphere of *Venus* 3636104 miles, hence the Circuit of her Sphere is 2285591 miles. Her mean Diurnal Motion is 1 *d.* 36'. 8". so that she moves in a day 101712 miles, in an hour 4238, in a minute 70.

4. The

4. The Earth accomplishes her Revolution in 365 dayes, 5 hours, and 59 minutes, which according to *Copernicus* Systeme is placed betwixt the Orbs of *Mars* and *Venus*. It is from the Sun to the Body of the Earth 5021896 Miles. The circuit of her Sphere is 31560207 miles, her Diurnal motion 39 minutes, 8 seconds. A Degree of a great Circle upon the Earths Superficies is commonly reputed 60 miles, but by Mr. *Norwoods* experiment is found to be 69 miles.

5. *Mars* performs his Revolution about the Sun in one year, 321 dayes, 22 hours, and 20 minutes. It is from the *Sun* to the Body of *Mars* 7635292 miles. The Circuit of its Sphere is 47993264 miles, his Diurnal motion 31 minutes, 27 seconds, so that he wheels in a day 69842 miles, in an hour 2910 miles, in a minute 48½ miles.

6. *Jupiter* runs his course in 11 *Egyptian* years, 315 days, 14 hours, and 30 minutes. It is from the *Sun* to *Jupiter* 26179152 Miles. The Circuit of his Sphere is 164554670 miles, and his Diurnal motion about the *Sun* is 4 minutes, 59 seconds. Hence he wheeleth every day 17996 miles, every hour 1583 miles, and every minute 26 miles.

7. *Saturn* performeth one Revolution about the *Sun* in 29 *Egyptian* years, 162 dayes,

dayes, 1 hour, 58 minutes. It is from the *Sun* to *Saturn* 47833576 miles, the Circuit of his Sphere is 300668192 miles, his proper dayly motion is 2 minutes, 0 seconds. Therefore he wheeleth in a day 1595[illegible] miles, in an hour 1498, in a minute 2[illegible] miles. About his Body is a bright flat Ring which encompasseth him about.

The *Moon* is a secondary Planet, and retains the Earth for her Center, about which she performs her Revolution in 27 dayes, 7 hours, 43 minutes. It is from the Center of the Earth to the Moon 20323[illegible] Miles. The Circumference of her Sphere is 1277483 miles. Her Diurnal Motion is 13 degrees, 10 minutes, 35 seconds, so she wheeleth about in a day 46757 miles, in an hour 1948 miles, and in a minute $30\frac{1}{2}$ miles.

The Fixed Stars have a Motion or Circumrotation upon their own Axis, and besides a Motion of Revolution from West to East, which in a year is not less than 49 seconds, nor greater than 51 seconds, that it seems most probable that their Annual Motion is 50 seconds, 40 thirds, whence it follows that they compleat not one Degree in the Ecliptick sooner than in 71 years, and $\frac{16}{104}$ or 19 dayes, and 12 hours in a manner, but the whole Circle of 360 degrees they run not through in less than 2557[illegible]

Siderea

Sidereal years, which is *Annus Magnus Platonicus*, (though by the Antients computed to extend to 36000 years.)

But according to the Opinion and Observation of the Learned *Ticho Brahe*, as you may find in *Mathematical Recreations*, *fol.* 220. Their Distances are thus.

From the Center of the Earth	to the	Moon	is near	56 Semidiameters or 192416 miles
		Sun		1142 Semidiam. or 3924912 miles.
		Starry Firmament		14000 Semidiam. or 48184000 mil.

But according to Mr. *Dees* Computation 2008 [illegible] Semidiameters, or 69006540 Miles: From whence these Corollaries do follow.

1. Suppose that a man should go 20 Miles in Ascending towards the Heavens every day, he should be above 15 years before he could attain to the Orb of the Moon.

2. If a Milstone should descend from the place of the Sun 1000 Miles every hour, (which is above 15 Miles in a Minute, far beyond the proportion of Motion) it would be above 163 dayes before it would fall down to the Earth.

3. The Sun makes a greater way in one day than the Moon does in 20 days, because that the Orb of the Suns Circumference is at the least 20 times greater than the Orb of the Moon.

And thus much shall suffice to have spoken concerning the Number, the Names, the Order, the Motions, Magnitudes and Distances of these Heavenly Bodies, the *Planets*, and *Fixed Stars*.

CHAP. VI.

Of the Spheare.

I Come now to speak of the *Spheare*, and of Questions of the *Spheare*, which I make the sum and substance of *Astronomy*, as it stands in Analogy with *Geometry* in this place: For as *Surveying* is taking Heights and Distances at Land, *Navigation* at Sea, and are performed by the Resolution of *Plain Triangles*: So this part of *Astronomy* is taking Heights and Distances in the Heavens (as is before hinted) and is performed by the Resolution of *Sphærical Triangles*, or by *Projection of the Sphear* it self, which is the ground of these *Spherical Triangles*. And that I may proceed the more metho-

methodically, and be more clearly understood.

1. I shall first shew you what these Questions of the *Spheare* are, and how they may be termed taking Heights and Distances.

2. I shall shew you how these Questions may be resolved.

I. By Trigonometrical Calculation, or the Resolution of Spherical Triangles.

II. By Projection of the Sphear: 1. More Natural, By the Globe it self. Or, 2. More Artificial, By Projection of the Spheare in *plano*, and so shew you these Questions may not only be resolved by the Globe, but also by *Planispheres*, *Quadrants*, &c.

III. And lastly, I shall insist more largely concerning that principal and most useful question of the Sphear, to wit, Dialling, or of finding the Hour of the Day by the Sun, and the Hour of the Night by the Moon and Stars, and shall herewith conclude the whole work.

SECT. 1.

First then, The Astronomical *Questions of the Sphear are these,*

1. *For Heights.* To find

1. The Height or Elevation of the Pole, or Latitude of the place.

 2. The

2. The Altitude or Height of the Sun or Stars above the Horizon at any time, which are called (particularly their Almicantaras, and their Meridian Altitude.)

2. *For Distances.*

3. The right Ascension of the Sun or Stars may be also called an Height, because it is the Meridian Altitude, or Height of the Equinoctial above the Horizon: But may be also called a Distance, because the Distance or Number of Degrees contained betwixt the first Degree of *Aries*, and the Degree of the Equinoctial that comes to the Meridian with the Sun or Stars is the Right Ascension of the Sun or Stars in Degrees and Minutes.

4. The Oblique Ascension of the Sun or Stars, though it sounds like an Height, is more properly a Distance, because it is the Distance in Degrees of the Equinoctial betwixt the first point of *Aries*, and that very point of the Equinoctial that rises or cuts the Horizon with the Sun or Stars.

5. The Ascensional Difference is in plain *English* no more but the Distance betwixt the Oblique and Right Ascension.

6. The Suns Meridian Distance North or South from the Equinoctial, or of a star from the Ecliptick is called their Declination.

7. The

7. The *S*uns distance from any of the 12 signs is called his place in the Zodiack.

8. His Distance of rising or setting from the due point of East or West, is called his Amplitude.

9. The *S*uns Distance at any time from any point of the Horizon is called his *Azimuth*, or Angle of the *Suns* position.

10. The *Suns* distance from the next Equinoctial point *Aries* or *Libra*, requires no Explanation.

Lastly, His Distance from the Meridian shews the hour of the day, and the Moon and stars distance from the Meridian shews the hour of the night. And all these in these respects may be properly called Heights and Distances in *Astronomy*.

SECT. 2.

Therefore in the second place I shall shew you how to find these Heights and Distances.

1. By Trigonometrical Calculation, or the Resolution of Sphærical Triangles.

2. By Projection of the Sphear, either Globular, or in *plano*, in whole, as in the Globe it self and Planispheares; or in part, as in Semicircles, Quadrants, and the like.

For the first of these, to shew you how these Questions may be resolved by the Resolution of Spherical Triangles, I shall endeavour to do these two things.

1. I shall speak something concerning Spherical Triangles, and the way how to make and measure them.

2. I shall give you the Trigonometrical proportions how to find these several Astronomical Heights and Distances before-mentioned by Sines and Tangents.

SECT. 3.

Of Spherical Triangles.

As *Plain Triangles*, so *spherical* consist of six parts, *viz.* three Angles, and three sides; the Angles of both are measured and protracted by a line of Chords. The difference betwixt them lies only in this, that the sides of Plain Triangles are Right lines drawn by the fiducial edge of a Ruler, and measured by a Line of equal parts: whereas the sides of spherical Triangles are Arches of great Circles (that is Arches of any Circle to the same Radius) protracted upon the Center of the same Arches or Circles, and measured by a Line of Chords after the same manner, that all Circles and Arches are measured by Degrees and Minutes.

And

And as the Proportions betwixt the sides and Angles of Plain Triangles are found by numbers and sines: As for Example, The Artificial sine of any Angle is proportionable to the Logarithmetical Number of its opposite side; and on the contrary, the Logarithmetical Number of any side is proportional to the Artificial sine of its opposite Angle: So are the sides of spherical Triangles (being Arches of great Circles; and the Angles also measured at their Quadrantal Distance, or at the Distance of the Radius from the Angular point,) and the proportion between these sides and Angles are found by sines only by this Rule. *Harm. Cœl. fol.* 10.

In every, or in any spherical Triangle the sines of the sides are proportional to the sines of their opposite Angles, and the sines of the Angles are proportional to the sines of their opposite sides.

And in these particulars lies the difference betwixt plain and spherical Triangles, and betwixt the protracting or making, and the measuring of the same: More particularly,

1. *To measure or take the Quantity of any angle of a sphærical Triangle, and to protract the same by the Projection.*

Lay a Ruler to the Angular point, and

the Extremity of the sides containing the Angle, they being continued to Quadrants, and note where the Ruler cuts the Meridian or outward Circle, at both which places make marks upon the Meridian: The Distance between those two marks measured upon your Line of Chords, shall give you the quantity of the Angle required.

And to Protract an Angle, do thus.

First, Taking 90 degrees or Radius from your Line of Chords, describe a Circle or part of a Circle, so much of it as you have occasion for, and making the Center to be the Angular point, prick out in the circumference so many Degrees and Minutes from the same Line of Chords as you would have your Angle to contain: Then at the distance of the Radius from these marks in the same Circumference placing one foot of your Compasses, with the other strike an Arch through the mark and the Center, do the like with the other mark: These Arches at the Center shall include an Angle of a sphærical Triangle, containing the same Number of Degrees as were markt out in the Circumference. The like Ratio holds in any other Angle. But,

2. To

2. To Measure, or take the quantity of any side of a Spherical Triangle,

And to protract the same, is more difficult, and requires in the first place (seeing that all Circles have their proper Poles) That the Poles of those Circles (whereof those sides are Arches) be found out.

3. Now to find out these Poles.

Note that the Pole of every great Circle is 90 degrees, or a Quadrant of a Circle and distant from the Circle it self upon that line which cutteth the Circle at Right Angles. Thus in a Projection of the Sphear the Poles of all the hour-Circles are upon the Æquinoctial, and the Poles of all the Azimuths upon the Horizon: The Poles of the Æquinoctial are the Poles of the World; and the Poles of the Horizon, the Zenith and Nadir. Now to find the Pole of any Arch continued to a Quadrant, lay a Ruler by the extream ends of the said Arch, and mark where it cuts the Meridian or outward Circle: Then take 90 Degrees from your line of Chords, and set off that distance from the said mark; and lastly, lay a Ruler from the former end of the Arch to this latter mark, and where this crosses the Circle

which cuts the Arch at right Angles, there is the Pole of the ſaid Arch. To make it more plain by an Example.

Suppoſe you would find the Pole of the Hour-circle PDS in the Projection, lay a Ruler upon P and D, it will cut the Meridian or outward Circle in *e*. Then take 90 degrees of your Line of Chords, and ſet them from *e* to *f*. A Ruler layd from P to F will cut the Æquinoctial in *y*, ſo is *y* the Pole of the Circle PDS, and ſo of any other as the Pole of ZGN is ☉, *&c.*

4. *Thus having found the Pole of a Circle, or of the ſide of a Spherical Triangle, which is the Arch of a Circle, To meaſure the ſaid ſide.*

Lay a Ruler upon the ſaid Pole, and to the extream ends of the ſide of a Triangle, note where the Ruler ſo laid cuts the Meridian or outward Circle at both ends of the ſide: That Diſtance taken in your Compaſſes, and meaſured upon the Line of Chords, will give you the Quantity of the ſide of the Triangle. As for Example in the Projection: Let it be required to find the ſide EZ of the Triangle ZEP, lay a Ruler to ☉, (the Pole of the Circle ZEN,) and the Angular point E, it will cut the Meridian in M, and a Ruler laid to Z will cut

the

the Meridian in Z, so the distance MZ taken in the Compasses, and measured upon the Line of Chords, will be found to contain 78 degrees, and such is the quantity of the side ZE.

5. *Now to Protract a side of a Spherical Triangle, is taught by measuring of it.*

For first you are to find the Pole, then laying your Ruler from the Pole by the extremity of one end of the side, make a mark upon the Meridian or outward Circle, and make another mark at the extremity of the other end of the side; and lastly find out the Center of the Circle to the same Radius, which will cut both the extremities of the side required, and it is done.

SECT. 4.

Having thus shewn you how to protract and measure either Angle or side of a Spherical Triangle, and withal given you *pag.* 343 the Rule to find out the proportions; or by some of the 6 parts given, to find the rest, *viz.* That the sines of their sides are proportional to the sines of their opposite Angles & *e contra*, which Rule is so general, that thereby you may resolve all, or most questions of the Sphere or Spherical

Trian-

Triangles: yet I think it not amiss to lay down the particular proportions by which these several Astronomical Questions of the Sphear are resolved and found by the lines of Artificial Sines and Tangents, or Logarithmetical Arithmetick.

How to find the Suns place
And his Declination,
His Meridian Altitude, and thereby,
The Latitude.

All these you will find Taught in *Navigation, pag.* 254, 257, and 258, whereunto I refer the Reader: They being there handled as they are *Data*, or things given: And therefore some of these being given, I shall shew you here how they and other postulata's or quæsita's may be found in these following questions.

PROBL. 1.

By the Suns shadow to find his Altitude.

Take a two-foot Rule, or any other staff divided into equal parts: And as the length of the shadow is to the length of the Rule or Staff (held perpendicular to the Horizon) upon the line of Numbers; so is the Tangent of 45, or Radius to the Suns Height required: As suppose the staff divided into 100 parts, the shadow is 83 of these

these parts; the extent betwixt 100 and 83 in the Line of Numbers will reach from Radius, or 45 upon the Line of Tangents to 30 *d.* 18 *m.* 26 seconds, the Suns Height above the Horizon.

PROBL. 2.

To find the Suns Declination. vide pag. 258.

PROBL. 3.

To find the Suns place you have also 257, But shall add this other Analogy, *The Suns greatest Declination, and his present Declination given, to find his Distance from the next Equinoctial point, and thereby his place in the Ecliptick.*

As his greatest Declination (which is alwaies 23 *d.* 30 *m.*) is to Radius or sine 90, so is his present Declination (which is also to be given) to his Distance from the next Equinoctial point: Suppose the Declination 11 *d.* 30 *m.* his distance is 30 *d.* from the next Æquinoctial point, which is *Aries*, therefore his true place is just leaving *Aries*, and entring *Taurus*.

PROBL.

PROBL. 4.

The Suns greatest Declination, and Distance from the next Æquinoctial point given, To find his Right Ascension.

As the Radius is to the Tangent of his Distance, *&c.* so is the Co-sine of his greatest Declination to the Tangent of his Right Ascension. Example, As 90 *d.* is to 66 *d.* 30, so is Tangent of 30 *d.* to the Tangent of 27 *d.* 50 minutes.

PROBL. 5.

The Suns greatest Declination, and his present Declination given, to find his Right Ascension.

As the Tangent of his greatest Declination (23 *d.* 30 *m*) is to Radius, so is the Tangent of his present Declination (which suppose 11 *d.* 30 *m.*) to the Sine of his Right Ascension 27 *d.* 50 *m.*

PROBL. 6.

To find the Oblique Ascension.

If the Declination be North, substract the Ascensional Difference from the right Ascension, and it will give the Oblique Ascension. If the Declination be South, add

add the ſame Aſcenſional Difference and Right Aſcenſion together, and the Sum will be the Oblique Aſcenſion.

PROBL. 7.

To find the Aſcenſional Difference, the Latitude and Declination given.

As the Co-Tangent of the Latitude, (which ſuppoſe 38 *d.* 30 *m.* the Latitude being 51 *d.* 30 *m.*) is to the Tangent of the Suns Declination (20 *degrees*) ſo is the Radius (90 *d.*) to the ſine of the Aſcenſional Difference (27 *d.* 14 *m*) and ſo much the Sun riſeth or ſetteth before or after ſix, according as the Declination is North or South, and this 27 *d.* 14 *m.* converted into time (by allowing 15 degrees to an hour, and one degree for 4 minutes of time) is 1 *h.* 49 *m.* and ſo much doth the Sun riſe or ſet before or after the hour of ſix, according to the time or ſeaſon of the year.

PROBL. 8.

The Latitude of the Place, and Declination of the Sun given, To find his Amplitude.

As the Co-ſine of the Latitude (38 *d.* 30) is to Radius (90.) ſo is the Sine of the Declination

clination (ſuppoſe 20 *d.*) to the ſine of the Amplitude (33 degrees 20 minutes.)

PROBL. 9.

To find the Suns Altitude, and firſt his Meridian Altitude, his Declination (only) given.

If the Declination be North, add the ſame to the Complement of the Latitude, and the Total of both ſhall be the Meridian Altitude. If the Declination be South, ſubſtract the ſame from the Complement of the Latitude, and what remains ſhall be the Meridian Altitude.

PROBL. 10.

The Latitude and Declination given, to find the Suns Altitude at 6 a Clock.

As the Radius (90 *d.*) is to the ſine of the Suns Declination (20 *d.*) ſo is the ſine of the Latitude (51 *d.* 30 *m.*) to the ſine of the Suns Altitude at 6, *viz.* 15 *d.* 30 *m.*

PROBL. 11.

The Latitude and Declination given, To find the Suns Altitude when he is upon the true Eaſt and Weſt points.

As the ſine of the Latitude (51 *d.* 30 *m.*) is to the ſine of the Declination (20 *d.*) ſo is the Radius (90 *d.*) to the ſine of the Suns

Suns Altitude, being due East or West (25 *d.* 55 *m.*)

P R O B L. 12.

To find the Suns Altitude at any time assigned.

First, Say as the Radius is to the Cotangent of the Elevation, so is the sine of the Suns distance from six to the Tangent of an Ark, which being substracted out of the Suns distance from the Pole. I say again, Secondly, As the Co-sine of the Ark found is to the Co-sine of the Residue of the Suns Distance from the Pole, so is the sine of the Elevation to the sine of the Height required.

P R O B L. 13.

To find the Angle of the Suns Position, having given the hour from Noon: The Latitude, and the Suns Altitude.

As the Co-sine of the Suns Altitude, (which suppose 78 *d.*) is to the sine of the hour from Noon (35 *d.* 36 *m.*) so is the Co-sine of the Latitude (38 *d.* 30 *m.*) to the sine of the Angle of the Suns Position at the time of the Question, (21 *d.* 45 *m.*)

The

PROBL. 14.

The Latitude of the place, and Declination of the Sun being given, To find the Angle of the Suns position at his Rising.

As the Co-ſine of the Declination (70 *d.*) is to Radius (90 *d.*) ſo is the ſine of the Latitude (51 *d.* 30 *m*) to the ſine of the Angle of the Suns Poſition at the time of his riſing.

PROBL. 15.

Having the Suns Altitude, His Declination, and Diſtance from the Meridian (or the hour of the day) given, to find his Azimuth.

As the Co-ſine of the Suns Altitude, (which ſuppoſe 64 *d.* 4 *m.*) is to his diſtance from the Meridian *(61 d.)* or 4 hours, 4 *m.* before noon (*viz.* 7 hours 56 *m.*) ſo is the Co-ſine of the Declination (78 *d.* 30 *m.*) to the Suns Azimuth from the South Eaſtwards, (*viz.* 72 *d.* 22 *m.*)

PROBL. 16.

The Latitude of the place and Declination of the Sun given, To find what Azimuth the Sun ſhall have at ſix a Clock.

As the Co-ſine of the Latitude (38 *d.* 30 *m.*) is to Radius (90 *d.*) ſo is the Co-tangent

Tangent of the Declination (70 *d.*) to the Tangent of the Suns Azimuth counted from the North part of the Meridian (77 *d.* 14 *m.*)

PROBL. 17.

The Declination, Altitude, and Azimuth of the Sun being given, To find the Hour of the Day.

As the Co-ſine of the Declination (78 *d.* 30 *m.*) is to the Co-ſine of the Altitude (64 *d.* 4 *m.*) ſo is the Azimuth (72 *d.* 22 *m.*) to 61 *d.*) the hour of the day. This 61 *d.* converted into time gives 4 hours, 4 minutes, which taken from 12 hours, the Remainder is 7 hours, 56 minutes before Noon: The time of the day required.

Mr, Leyburns *Analogy is this,*

As the Co-ſine of the Declination (70 *d.*) is to the ſine of the Azimuth (146 or 34 degrees) ſo is the Co-ſine of the Altitude (78 *d.*) to the ſine of the hour from Noon 35 *d.* 36 *m.*

CHAP.

CHAP. VII.

THus you have the several Analogies and Proportions, whereby may be resolved these most useful Questions or Problems of the Sphear by the *Rule of Three*, wrought either by *Logarithmes*, *pag.* 31. or by the Lines of Artificial Numbers, Sines, and Tangents, *pag.* 41. taking this Observation along with you.

That (*As*) denotes, or is put before the first Number: (*Is to*) the second: (*So is*) denotes the third: and (*To*) denotes or precedes the fourth, so that multiplying the second by the third, and dividing the Product by the first, produceth the fourth in the Direct *Rule of Three*, *pag.* 19. and multiplying the First by the Second, and dividing by the Third, produceth the Fourth by the Indirect or Reverse *Rule of Three*, *pag.* 19. where you may find both the said Rules.

Further you may observe, That the Suns Declination and Altitude is Assistant to find the Latitude: His Amplitude and Azimuth, to find the variation of the Compass: The Ascensional Difference to find the time of the Suns Rising and Setting: The right Ascension

ſcenſion of the Sun and Stars is Aſſiſtant find their coming upon the Meridian, d the hour of the Night. The Latitude, eclination, and the Suns Altitude, and zimuth is Aſſiſtant and ſubſervient to the nding the hour of the day: Theſe two ſt put together, that is, the finding of e hour of the Day and Night is that hich we call Dialling, concerning which, ou will find ſomething particularly in the Concluſion of this ſmall Treatiſe.

Having now ſhown you how thoſe queſtions of the Sphear may be reſolved by *Trigonometrical Calculation*, and their ſeveral Analogies and Proportions alſo, by which they may be found by Arithmetical Operation. I come now to ſhew you how theſe may be done by ſeveral Projections.

Firſt, By the Globe it ſelf, being the moſt natural projection of the Sphear of all others whatſoever.

Secondly, By *Theakers Planiſphear*, an Inſtrument of good uſe for this purpoſe, called a light to the Longitude.

Thirdly, By Mr. *Foſters* Quadrant, that is a Quadrant with hour Lines.

Fourthly, By the Projection of the Sphear in plano. And,

Laſtly, I ſhall particularly inſiſt more at large upon that Queſtion concerning finding of the hour of the day by the Sun, and

of

of the hour of the Night by the Moon an Stars, which may be properly termed D alling, and with that conclude what I he intend in this Manual.

SECT. I.

The Use of the Globe in questions of the Sphea

Those who have a mind to be through instructed in the general Use of the Globe both Cœlestial and Terrestrial, I recommen to them that most compleat Book writt by Mr. *Joseph Moxon* concerning the Use the Globes in the solution of Astronomica Geographical, Nautical, Astrologica Gnomonical Problems, and in the Resol tion of Spherical Triangles at large: Wh I here intend is after a very brief manne to shew you how these Questions of t Sphear may be resolved by the Globe ea and speedily.

First then, You are to find the Latitude the place where the question is propound

Which to do by Observation is taug *pag.* 255. but if you desire to do it by Globe, you are to bring the particular pl to the Brazen Meridian, and the degr reckoned thereupon from the Equator

the place whose Latitude you seek, is the Latitude whether North or South: And for the Longitude likewise you may find it by Observation, as in *pag.* 260. and by the Globe. Observe what Degrees of the Equator comes to the Meridian with the place, the same is the degrees of Longitude from the first Meridian.

In the next place you are to rectifie your Globe fit for use: Which to do,

1. When you rectifie the Globe to any particular Latitude, you must move the Brazen Meridian through the Notches of the Horizon, till the same Number of Degrees accounted on the Meridian from the Pole (towards the North point in the Horizon, if in North Latitude, and towards the South point, if in South Latitude) come just to the edge of the Horizon.

2. Next rectifie the Quadrant of Altitude, and screw the edge of the Nut that is even with the graduated edge of the thin plate to the same Number of Degrees as the Latitude is accounted on the Meridian from the Equinoctial, which will be the Zenith of the place.

3. Bring the degree of the Ecliptick the Sun is in that day to the Meridian, (which you shall learn to know hereafter) and then

then turn the Index of the Hour-Circle to the hour of 12 on the South-side the Hour-Circle, if in North Latitude & *contra*; and then is your Hour-circle also rectified, fit to use for that day.

Lastly, To rectifie the Globe to correspond in all respects with the position of the Sphear, you must with a Needle placed in the bottom set the 4 quarters of the Horizon *East*, *West*, *North*, and *South*, agreeable to the 4 Quarters of the World, and the Horizon by a Level you may set parallel to the Horizon of the World.

And positing the degree of the Ecliptick the Sun is in to the Height above, or depth below the Horizon the Sun hath in Heaven: Your Globe is made correspondent in all points with the frame of the Sphear, for that particular Time and Latitude.

The Globe thus rectified, you may find all the forementioned Problems on this manner.

1. Find the day of the Moneth in the wooden Horizon, and over against it you will find the degree of the Ecliptick, which the Sun is in that day: As suppose at *London* (where the Pole is elevated 51½ degrees of Northern Latitude, the Globe rectified for the same Latitude, as also the Hour-Circle, and Quadrant of Altitude) over

again

against the 10*th* of *May* you will see the Suns place in the Zodiack is 29 degrees of ♉ *Taurus.*

2. Bring this degree which is the Suns place to the East-side of the Horizon, and its distance from the Equinoctial, or from the due East point is the Suns Amplitude, or so much as the Sun riseth before or after the hour of six 33 *d.* 20 *m.* from the East-point Northwards in this Example.

3. Look upon the Hour-Circle, and the Index will point to the hour of Sun-rising, which in this Example is 12 Minutes after 4 in the morning.

4. Observe what degree of the Æquinoctial riseth with the Suns place, and that is his Oblique Ascension, reckoning from the Vernal colure in the Cœlestial, and from the first Meridian in the Terrestrial Globe.

5. As you move the Globe round upon his Axis Westward, you may (at any time or hour which the Index of the Hour-Circle points to) know the Suns Altitude at that time, by bringing the Quadrant of Altitude to cut the Suns place: As at 53 *m.* past 8 in the morning, you will find it just 40 degrees high above the Horizon: And,

6. The Quadrant of Altitude at the Horizon will shew his Angle of Position, or upon what point of the Compass it is.

7. The Suns Azimuth is the ſame with his Angle of Poſition.

8. And his Almicanter the ſame with his Altitude.

9. Bring the Suns place to the Meridian, and there the degrees intercepted betwixt the Suns place and the Horizon, reckoning upon the Brazen Meridian, is the Suns Meridian Altitude: In this Example, (50 *d.* 30 *m.*)

10. The Diſtance betwixt the Suns place in the Ecliptick, and the Equinoctial is the Suns Declination reckoned alſo upon the Meridian: In this Example (20 *d.* 5 *m.*)

11. The Degree of the Equinoctial which comes to the Meridian with the Suns place, is his right Aſcenſion, reckoning from the Vernal Colure, or firſt Meridian.

12. The number of degrees comprehended betwixt the Oblique and Right Aſcenſion, is the Aſcenſional Difference reckoned upon the Equinoctial.

13. At any Degree of Altitude, either before Noon, or after Noon, the Index of the hour-Circle points to the hour of the day.

14. And laſtly, when the Suns place comes to the Weſt ſide of the Horizon, the Index of the Hour-Circle will point to the hour of Sun-ſetting, and alſo will ſhew his Oblique Deſcenſion, and upon what Azimuth

Azimuth or point of the Compaſs it ſets, and how much before or after ſix.

What is here ſaid of the Sun, the ſame is to be underſtood of any Star, uſing the ſaid Star as you do the Suns place: As to Inſtance in theſe particulars: The Globe rectified as before.

1. By the Meridian Altitude of any Star, to find the Elevation of the Pole: Obſerve by a Quadrant or other Inſtrument the Meridian Altitude, and bringing the Star to the Meridian, place it upon the Globe to the Altitude obſerved, ſo ſhall the Number of Degrees intercepted between the Pole and the Horizon, be the elevation of the Pole.

2. To find the Right Aſcenſion of any Star, bring the Star to the Meridian, and the Number of degrees betwixt the Vernal colure, and the Meridian its Right Aſcenſion; in like manner for his Oblique Aſcenſion, or oblique Deſcenſion.

3. To find the Declination of a Star, bring the Star to the Meridian and count the Number of Degrees betwixt the ſaid Star, and the Equinoctial, the ſame is the Stars Declination be it North or South.

4. The Azimuth and Almicantara, or which is all one, The Altitude and Angle of Poſition of any Star is found the ſame way as the Suns Azimuth and Almicantara, as in 5, 6, 7, 8, *fol.* 369.

5. To know any Star in Heaven, Firſt, having a Theodolite, with a moveable Index, upon the Index fix a two-foot Rule, or ſuch a thing perpendicular; and at the top of the Rule hang a Quadrant, then placing your Theodolite trueNorth and South by help of a Needle, or a Meridian Line, you may by the Quadrant take the Almicantara or Altitude; and the Index of the Theodolite will ſhew the Azimuth or Angle of Poſition: And obſerve by the Globe what Star there hath the ſame Azimuth and Almicantara at the ſame time, and that is the ſame Star which you obſerved in Heaven: For if every Star on the Globe had a hole in the midſt, and your Eye were placed in the Center of the Globe, you might by keeping your Eye in the Center, and looking through any Star on the Globe, ſee the ſame Star in the Heaven, which that on theGlobe doth repreſent; for from the Center of the Globe there proceeds a ſtrait Line through the Star on the Globe even to the ſame Star in Heaven.

6. The Meridian Altitude of a Star is found upon the Globe by bringing the ſaid Star to the brazen Meridian, and reckoning how many degrees it is above the Horizon, the Globe rectified as before ſhown.

7. And by the Hour-Circle, turning about the Globe, you may obſerve at what

hour

hour any Star will be upon the Meridian, to wit, when the same Star on the Globe touches the Brazen Meridian.

8. The Globe rectified, and the Altitude of a Star found by a Quadrant in the Heavens, and the same Star brought to the same degree of Altitude upon the Globe, by the Quadrant of Altitude, the Index of the hour-Circle will point to the hour of the night.

9. The Rising and Setting of the Stars are found as the rising and setting of the Sun: But this rising and setting admits of a threefold distinction:

1. Cosmical, when any star riseth with the Sun, it is said to rise Cosmically, and when any star sets when the Sun riseth, it is said to set Cosmically.

2. *Acronical*, The stars that rise when the Sun sets are said to rise Acronically, and the stars that set with the Sun are said to set Acronically.

3. *Heliacal*, when a star formerly in the Suns Beams gets out of the Sun Beams, it is said to rise Heliacally, and when a star formerly out of the Suns Beams gets into the Suns Beams, it is said to set Heliacally.

And what stars do rise or set any of these waies, and when may be easily found by the Globe, as in *Moxons lib.* 2. *Probl.* 35, 36, 37, which I shall not here insist upon.

Lastly, I shall shew you how to find the Longitude and Latitude of a star, and so conclude as to the Cœlestial Globe.

If the star you enquire after be on the North-side the Ecliptick, you must elevate the North Pole 66½ degrees: If on the South-side you must elevate the South Pole as much, then bring the solstitial colure to the Meridian on the North side the Horizon, and screw the Quadrant of Altitude to the Zenith, which will be 23½ degrees from the Pole of the World, so shall the Ecliptick lye in the Horizon, and the Pole of the Ecliptick also lye under the Center of the Quadrant of Altitude: Now to find the Longitude of any star, turn the Quadrant of Altitude about till the graduated edge of it lye on the star, and the degree in the Ecliptick that the Quadrant touches is the Longitude of that star: And the degree of the Quadrant of Altitude that touches the star is the Latitude of the star, so the Longitude of *Marchab* a bright star in the Wing of *Pegasus* will be found ♓ 18 *d.* 56. and its Latitude 19 *d.* 26 *m.*

What hath hitherto been said, concerns the Cœlestial Globe, I shall add a few propositions proper to the Terrestrial, which together will contain the principal Problems to be performed by the Globe.

To

1. *To find the Distance between any two places on the Terrestrial Globe.*

The graduated edge of the Quadrant of Altitude applyed to both the places, will shew the number of Degrees betwixt them; or with a pair of Compasses pitch one foot in one of the places, and the other foot in the other place, and keeping your Compasses at that distance, apply the feet to the Equinoctial Line, and you will find the number of Degrees comprehended between them: As suppose the places were *London* and *Jamaica*, you will find 68 $\frac{1}{2}$ degrees, which multiplyed by 60, makes 4110 *English* Miles.

2. *To find upon what point of the Compass any two places are scituate one from another.*

Find the two places on the Terrestrial Globe, and see what Rumb passes through them, or if no Rumb pass immediately through both, take that Rumb which runs most parallel to both the places.

3. *To know what a Clock it is in any other part of the Earth.*

Note that those places to the Eastward

of us have their day begin ſooner than ours, and thoſe to the Weſt-ward later; therefore firſt bring the place of your own habitation to the Meridian, and the index of the Hour-Circle to 12, then bring the other place to the Meridian, and the Index will point to the hour in the other place, be it ſooner or later.

4. *Any place on the Terreſtrial Globe given, to find its Antipodes.*

Bring the given place to the Meridian, ſo you may find its Longitude and Latitude, then turn about the Globe till 180 degrees of the Æquator paſs through the Meridian, and keeping the Globe to this poſition, number on the Meridian 180 degrees from the Latitude of the given place, and the point juſt under that degree is the Antipodes; or bring the given place to the North or South point of the Horizon, and the point of the Globe denoted by the oppoſite point of the Horizon is the Antipodes of the given place.

5. *To find the Perecy of any given place.*

You muſt underſtand the Perecy of any place are ſuch as dwell about the ſame Hemiſphere in the ſame Parallel from the Æquator, the one Eaſtern, the other Weſtern, directly oppoſite to one another: There-

Therefore bring your place to that side the Meridian which is in the South notch of the Horizon, and follow the parallel of that place on the Globe till you come to that side the Meridian which is in the Northern notch of the Horizon, and that is the Perecy of your place.

7. *To find the Antecy of any given place.*

The Antecy are such as dwell one against another, having the self same Meridian, and equal distance from the Æquator, the one in the Northern, the other in the Southern Hemisphere, therefore bring your place to the Meridian, and find its Latitude: Then if it have North Latitude, count the same number of Degrees on the Meridian from the Æquator South-wards: but if it have South Latitude, count the same number of Degrees from the Æquator Northwards, and the Point of the Globe directly under that number of Degrees is the Antecy of your place.

8. *How to hang the Terrestrial Globe in such a Position, that by the Suns shining upon it, you may with great delight at once behold the Demonstration of many Principles in Astronomy and Geography.*

Take the Terrestrial Ball out of the Horizon, and fasten a Thread on the Brazen

Meridian to the Degree of the Latitude of your place: By this thread hang the Globe where the *Sun*-beams may have a free access to it: Then direct the Poles of the Globe to their proper Poles in Heaven, and with a thread fastned to either Pole, brace the Globe so that it do not turn from his position: Then bring your habitation to the Meridian, so shall your Terrestrial Globe be rectified to correspond in all respects with the Earth it self, so that with great delight you may behold

1. How the counterfeit Earth (like the true one) will have one Hemisphear *Sun*-shine-light, and the other shadowed by the shining Hemisphere, you may see that it is day in all places that are scituate under it, for on them the *Sun* doth shine: And that it is Night at the same time in those places that are scituated in the shadowed Hemisphear; for on them the *Sun* doth not shine.

2. If in the middle of the enlightned Hemisphear you set a *Spherick* Gnomon perpendicularly, it will project no shadow, but shew that the *Sun* is just in the Zenith of that place, that is directly over the heads of the Inhabitants.

3. If you draw a Meridian Line from one Pole to the other just through the middle of the enlightned Hemisphear, in all places under that Line, it is Noon; In those places

ces scituate to the West it is Morning, for with them the Sun is East; and in those places scituate to the East, it is Evening, for with them the Sun is West.

4. So many degrees as the Sun reaches beyond either the North or South Pole, so many Degrees is the Declination of the Sun either Northwards or South-wards, and in all those places comprehended in a Circle described at the termination of the *Sun*-shine about that Pole, it is alwaies day till the Sun decrease in Declination; for the Sun goes not below their Horizon, as you may see by turning the Globe about upon its Axis: And in the opposite Pole at the same distance the Sun-shine not reaching thither, it will be alwaies Night till the Sun decrease in Declination, because the Sun riseth not above their Horizon.

5. If you let the Globe hang steddy, you may see on the East-side of the Globe in what places it grows Night; and on the West-side of the Globe how by little and little the Sun encroaches upon it, and therefore there makes it day.

6. If you make of paper or parchment a narrow Circle to begirt the Globe just in the Equinoctial, and divide it into 24 equal parts to represent the 24 hours of day and night, and mark it in order from 1, 2, 3, to 12, and then begin again with 1, 2, 3,

to other 12, you may by placing one of the Twelves upon the Equinoctial under the Meridian of your place have a continual Sun-dial of it, and the hour of the day given on it at once in two places, to wit, where the enlightned Hemisphere is parted from the shadowed both on the East and West-side of the Globe; much more might be said on this subject, but the ingenious Artist may of himself find out diversities of other speculations.

And thus much shall suffice to have spoken concerning the use of the Cœlestial and Terrestrial Globes, and so I shall return from this Digression to my former proposed Method; and shew you how most of those formerly mentioned questions of the Sphear, or Spherical Problems, *pag.* 398 may be resolved.

SECT. 2.
By Theakers Planisphear.

This Instrument consists of two Sphears or Plains, the lowermost and fixed, wherein is described three several hour-Circles, the uppermost divided into 24 hours, wherein is reckon'd the Right Ascension of the Sun or Stars in hours and minutes: The next is called *Aries* hour-Circle, having ♈ standing betwixt 24 of the outward hour-Circle, and 12 in the inward or third hour-

our-Circle; in this is reckon'd the time of
he Sun and ſtars riſing and ſetting, or
f the ſtars coming to the South: The third
our-Circle is divided into twice twelve,
nd in this is reckoned the difference of
ime, and next to this is the Æquinoctial
ircle divided into 360 degrees: In the
pper moving Sphear or Plain is drawn,
irſt, The Circle of Moneths, placing
Aries at the 10*th.* day of *March*, by which
neans you may there find any day of the
Ioneth you deſire: Secondly, Next to this
s again placed another Æquinoctial Circle
ivided into 360 degrees. Thirdly, The
cliptick, containing the 12 ſignes of the
odiack, each ſigne divided into 30 degrees.
There is alſo belonging to this moving
Plain a moving Meridian, I call an Index
raduated both waies from the Equinoctial,
whoſe length is equal to the Diameter of
he Æquinoctial deſcribed on the upper mo-
ving Plain: The other parts whereof be-
yond what is graduated ſerves to cut the
ircle of Moneths, the Equinoctial and
our Circles on the lower Plain as occaſion
s. Laſtly, There ought to be alſo a turn-
ing Horizon which may be fitted for any
Latitude, having two Indexes thereunto
belonging, a longer and a ſhorter according
to the Pattern, to make which Horizons
for all Latitudes upon a piece of Paſt-board,
draw

draw 2 lines perpendicularly crossing each other in the middle, the length of one of these Lines make equal to the Diameter of the upper Sphear, and upon the other Line from the Center mark out the Latitude, by which means there is found three points, now find the Center to those 3 points, and strike an Arch and cut out by that Arch the Horizon required.

PROBL. I.

How by this Instrument to find the Declination place in the Ecliptick, and the Right Ascension of the Sun or any Star.

First, Turn the upper Sphear till Aries ♈, or the 10*th* of *March* stands directly against *Aries* ♈ in the lower Sphear. Then bring the moving Meridian to the day of the Moneth, if your question be concerning the *Sun*; or if it concern a star, lay the Index upon the Center of the star (whereof you find many with their Names added upon the upper moving sphear.) This done, observe that the Distance betwixt the Æquinoctial on the upper plain, and the suns place or star; is the suns or stars Declination; and the point or place cut in the Ecliptick at the same time by the Index, shews the sign, degree, and minute, the sun or star is

in

in; being their place or Longitude in the Ecliptick: And the Degree and Minute then cut in the Equinoctial by the Index, ſhews the ſun or ſtars right Aſcenſion in degrees and Minutes, and the ſame is ſhewn in Hours and Minutes in the outermoſt Hour-Circle of the lower Plain: For Example, the 20*th* of *April* the ſuns Declination is 15 degrees North, his place is 10 *d.* 8 *m.* of ♉ *Taurus*, his Right Aſcenſion 38 *d.* which in time is two hours, 32 minutes.

The 17*th* of *June* the great Dogs declination, 16 *d.* 32 *m.* his place or Longitude 7 degrees of *Cancer* ♋: His Right Aſcenſion 98 *d.* and in time 6 hours, 32 minutes.

PROBL. 2.

To find the Riſing and Setting of the Sun or any Star.

If the ſun or ſtar have North Declination, bring the longeſt Index of the Horizon to 12 hours in *Aries* hour-Circle, which will alſo cut 20 degrees of Longitude in the Equinoctial, the Meridian of *London*: But if the ſun or ſtar hath ſouth Declination, then bring the ſhorteſt Index of the Horizon to the ſame place, and there keep the Horizontal Index faſt, and if you deſire to know its Riſing, turn the upper moving

ving ſphear till the place of the Sun or the ſtar come exactly to touch the Circular edge of the Moving Horizon in the Eaſt Semi-circle of the moving ſphear ; or if you deſire its ſetting, bring the Suns place or ſtar to touch the Horizon in the Weſtern Semi-circle : Then turn the moving Index or Meridian to the day of the Moneth, and the ſame Index will there ſhew you the time of the Sun or ſtars riſing or ſetting in *Aries* hour-Circle: For Example, the 20*th* of *April* the Suns place being 10 *d.* 12 *m.* of *Taurus*, riſeth at 4 *h.* 48 *m.* in the morning, and ſets at 7 *h.* 12 *m.* in the Afternoon. Alſo the great Dog the 10*th* of *March* riſeth at one a Clock 52 minutes in the Afternoon, and ſets at 11 hours, or 11 a Clock 8 minutes at night.

PROBL. 3.

To know when any Star comes to the South.

Firſt, Bring the Moving Meridian to 12 hours in *Aries* Hour-Circle, and 20 degrees in the Æquinoctial, the Longitude or Meridian of *London*: keeping that faſt, move the upper ſphear till the ſtar come juſt to the graduated edge of the moving Meridian, and there keep the ſtar faſt, then turn the ſaid moving Meridian to the day of the Moneth,

Month, and in *Aries* Hour-Circle the ſame moving Meridian will ſhew you the time of that ſtars coming to the South: So the great dog the 1*ſt*. of *January* comes to the South 11 a Clock at night; and *Orions* left ſhoulder the ſame day at 9 *h*. 37 *m*. &c.

To uſe Theakers *Planiſphere inſtead of a Nocturnal, or by it to find the hour of the Night by any Star which is thereon.*

This may be done two ſeveral waies: The firſt is the ſame with that deſcribed (376.) *Prob*. 3. for to know at what hour any ſtar comes to the South muſt neceſſarily ſhew what hour it is when that ſtar comes to the South, and conſequently any other hour may be found by knowing how many hours ſuch a ſtar is paſt the Meridian, or ſhort of it: And therefore,

1. Suppoſe 12 hours in *Aries* hour-circle, and 20 *d*. in the Equinoctial, the Meridian of *London* (which are both one) to be inſtead of the *Flower de Luce*, or North point in a Nocturnal: By help of the Index bring the ſtar on the Inſtrument ſo many hours paſt it, or ſhort of it, as the ſame ſtar is paſt or ſhort of the North-point in Heaven: Keeping the ſtar to that poſition, move the Index to the day of the Moneth, and in *Aries* hour-Circle the ſame Index will ſhew you the hour of the night. 2. Sup-

2, Suppose ♈ to be instead of the *Flower de Luce* or the North point, bring the star in a due position thereunto, and the day of the Moneth shall point exactly to the hour of the night in the third hour-Circle next below *Aries* hour Circle in the Fixed Plain: So the first of *April* the bright star in the Rump of the great Bears Tail being upon the Meridian, either of these waies it will appear to be ¼ of an hour past 11.

PROBL. 4.

To find at what time the Moon, or any of the Planets come to the South.

First, You must know the Sign and Degree in the Ecliptick which the Moon or Planet is in; and bringing the moving Meridian or Index to 12 hours and 20 degrees the Longitude of *London*, bring the degree which the Moon or Planet is in to the edge of the Meridian, and there keep both fast and then look in the Circle of Moneths, and right against the day of the Moneth in *Aries* hour-Circle, you will see at what Clock that Planet comes to the South: But for the Moon you must remember to add 2 minutes for each hour, *vide Theakers* Book, *pag*. 20.

Those that desire to know more of this Instrument

Instrument, I refer them to the ſame Book of *Theakers*, and ſo paſs on to ſhew how the moſt of theſe Queſtions of the Sphear may be found.

PROBL. 5.

As an Appendix to theſe things, Concerning the Moon, I ſhall ſhew you how to find the Moons place.

Multiply the Moons age by 2, and divide the Product by 5, the Quotient will ſhew how many Signs, and the Remainder or Fraction, ſo many times 6 degrees as the Moon is gone from that ſign and degree where the Sun is at that preſent: Or more readily by this following Table.

The

The Dayes of the Moons Age.

D.	S.	D.	M.
1	0	13	11
2	0	26	21
3	1	9	32
4	1	22	42
5	2	5	53
6	2	19	3
7	3	2	14
8	3	15	25
9	3	28	35
10	4	11	46
11	4	24	56
12	5	8	7
13	5	21	18
14	6	4	28
15	6	17	39
16	7	00	49
17	7	14	—
18	7	27	11
19	8	10	21
20	8	23	32
21	9	00	42
22	9	19	53
23	10	3	3
24	10	16	14
25	10	29	25
26	11	12	35
27	11	29	46
28	00	8	56
29	00	22	7
30			

The Hours of the Moons Age.

H.	S.	D.	M.
1	—	0	33
2	—	1	6
3	—	1	39
4	—	2	12
5	—	2	45
6	—	3	18
7	—	3	51
8	—	4	24
9	—	4	56
10	—	5	29
11	—	6	2
12	—	6	35
13	—	7	8
14	—	7	41
15	—	8	14
16	—	8	47
17	—	9	20
18	—	9	53
19	—	10	26
20	—	10	59
21	—	11	32
22	—	12	5
23	—	12	38
24	—	13	11

The use of the Table by an Example.

As suppose the 19*th* day of *June* 1675, at Noon, you would know what sign the Moon is in: The change was the 12*th* day, at 8 hours at night; therefore the 19*th* day at Noon the Moon is 6 dayes, 16 hours old.

The place of the Sun and Moon at the change was 3 signs, 1 degree, 0 *m.* the Moons motion for 6 dayes by the Table is 2 signs, 19 *d.* 3 *m.* and for 16 hours, 8 *d.* 47 *m.* the sum is 5 *s.* 28 *d.* 50 *m* that is in 28*d.* 50 *m.* of *Virgo*, which is the 5 sign from *Aries*: The like may be done at any other time. So much shall suffice to have spoken concerning the Moon.

SECT. 3.

By the Quadrant.

I shall first give you a Description of the Lines thereupon, and then shew you the uses thereof: The double Arch next the Center is only for Ornament: The next Arch is the Æquinoctial, and the next to that represents the Tropique; betwixt which two Arches is the Line of Declination upon the side of the Quadrant, containing 23 *d* 30 *m.* and within are the Azimuth or hour-lines: Also Crosswise betwixt the

the Æquinoctial and Tropique is the Ecliptick, containing three of the signs, each sign divided into 30 *d.* or at least supposed to be so: Next to the Ecliptick, and joyning with it in the Æquinoctial is the Horizon, divided, or supposed to be divided into 40 *d.* and ending or crossing the Tropick in 3; *d.* 9 *m.* Next below the Tropick is the Circle of Months, which you may know by the Letters prefixt: The next Arch below these is the Quadrat, or that which serves instead thereof, the line of right shadows, and contrary shadows numbred from 1 to 10, and so back again to 1.

And the lowest of all is the Limb of the Quadrant divided into 90 degrees, and each degree into halfs. For the use thereof,

1. The Thread laid on the day of the Moneth, and the Bead just to touch the hour-line of 12: at the same time the thread will shew in the Limb of the Quadrant the Meridian Altitude of the sun, and moving the Bead from one hour to another will shew the suns Altitude at any hour whereon the Bead resteth, and the Bead amongst the hour-lines will shew the hour of the day.

2. *How to find the Hour of the Night by some principal Stars set upon the Quadrant.*

How to place any star upon the Quadrant. First

Firſt, You muſt know the ſtars Declination from the Æquator, and his Right Aſcenſion from the next Æquinoctial Point: As the Declination of the Wing of *Pegaſus*, being 13 *d.* 17 *m.* His right Aſcenſion 358 *d.* 34 *m.* from the firſt point of *Aries*, or 1 *d.* 26 *m.* ſhort of it: If you draw an occult parallel through 13 *d.* 17 *m.* And then lay a Ruler to the Center A, and 1 *d.* 26 *m.* in the Quadrant, the point where the Ruler croſſeth the Parallel, ſhall be the place for the Wing of *Pegaſus*, and ſo for the reſt.

Stars.	R. Aſcenſion.	Declination.
Arcturus	30 *d.* 7 *m.*	21 *d.* 6 *m.*
Lyons Heart	32 *d.* 28 *m.*	13 *d.* 42 *m.*
Bulls Eye	64 *d.* 18 *m.*	15 *d.* 46 *m.*
Aquila	66 *d.* 26 *m.*	8 *d.* 3 *m.*

To find the Hour of the Night by theſe Stars.

Firſt, put the Bead to the ſtar which you intend to obſerve, take his Altitude, and note how many hours he is from the Meridian, then out of the right Aſcenſion of the ſtar take the right Aſcenſion of the ſun converted into hours, and mark the Difference; for this difference being added to the obſerved hour of the ſtar from the Meridian, ſhall ſhew how many hours the *Sun* is gone from the Meridian, which in effect is the hour of the Night: As for Example, the 9*th* of *July* the *Sun* being then in 26 *d.* ♋

of

of *Cancer*, I ſet the Bead to *Oculus Tauri*, or the Bulls Eye, and obſerving his Altitude, ſhould find him to be in the Eaſt about 12 degrees high, and the Bead to fall on the hour-line of 6 before noon, which is 18 hours paſt the Meridian, (or paſt 12 at Noon.) The hour of the Night would be better than a Quarter paſt two of the Clock in the morning. For 118 *d.* the Right Aſcenſion of the *Sun* converted into time, makes 7 *h.* 52 *m.* this taken out of 4 *h.* 15 *m.* the Right Aſcenſion of *Oculus Tauri* adding a whole Circle (or 24 hours) for otherwiſe there could be no ſubſtraction, the Difference will be 20 *h.* 23 *m.* and this being added to 18 hours, which was the obſerved Diſtance of *Oculus Tauri* from the Meridian, ſhews that the *Sun* (abating 24 hours for the whole Circle) is 14 *h.* 23 *m.* paſt the Meridian, and therefore 23 *m.* paſt two of the Clock in the morning.

3. At what hour the Bead toucheth the Line of Declination, that is, the hour of the *Suns* riſing or ſetting: And bring the Bead to the Horizon, and the degrees cut in the Limb is the Aſcenſional Difference.

4. The degrees cut by the Bead in the Line of Declination, ſhews the *Suns* Declination for that Day.

5. The Bead brought to the Ecliptick ſhews the *Suns* place.

5. Brough

5. Brought to the Horizon ſhews the Amplitude, which is alwaies North when the *Sun* is in Northern ſigns, and South when the *Sun* is in Southern.

6. The Bead ſhews the Azimuth, the thread being laid to the *Suns* Altitude.

7 The thread laid to the *Suns* place, it ſhews in the Limb of the Quadrant his right Aſcenſion.

8. Moving the Thread backwards from the Line of Declination towards the hour-line of 12: It ſhews in the Limb the degrees of the *Suns* Depreſſion under the Horizon at each hour, and conſequently when it is 18 degrees under the Horizon it ſhews the beginning of day break, and end of twilight.

Thus you ſee all theſe Aſtronomical Queſtions are eaſily and ſpeedily reſolved by the Quadrant, but obſerve that theſe Quadrants do ſerve only for ſome particular Latitudes.

As an Appendix I ſhall add the ingenious uſe of the Quadrat upon the Quadrant, and ſo conclude this Point: This which is called the Quadrat is ſupplyed in a Line drawn next above the Limb of the Quadrant numbred from 1 to 10 in the middle, and ſo back again to 1, (as in the Deſcription,) whereof the former part are called

Right ſhadows, and the latter to the right hand are called contrary ſhadows, and by theſe you may find an Height by a known Diſtance, or a Diſtance by a known Height. And for performing hereof take theſe Rules.

1. If the thread fall in a right ſhadow, As 100 is to the parts on which the Thread falleth, ſo is the Diſtance to the Height required: Or, As the parts cut by the Thread is to 100, ſo is the Height to the Diſtance required.

2. If the Thread fall in a contrary ſhadow, As the Parts cut by the Thread are to 100, ſo is the known diſtance to the Height required: Or, As 100 is to the parts cut by the Thread, ſo is the known Height to the diſtance required. To make this more plain, If the thred fall on 100 parts, which is denoted by 10 in the Quadrant, or on 45 degrees in the Quadrant; the Height is equal to the diſtance above the Level of your Eye: If it fall on 50 parts of Right ſhadow, the Height is half the diſtance, if it fall on 25, it is $\frac{3}{4}$ 3 quarters of the diſtance: But if the Thread fall on 50 parts of contrary ſhadow, the Height is double to the diſtance; If on 25, it is 4 times the diſtance, and the like of the reſt.

CHAP.

CHAP. VIII.

BUT to return to my propoſed Method, having ſhewn you how theſe Sphærical Problems or Queſtions of the Sphear may be eaſily and readily reſolved by thoſe Artificial Projections of the *Globe*, the *Planiſphear*, and *Quadrant*, and by way of Digreſſion, having added the other uſes alſo of theſe Inſtruments, I come now in the laſt place to ſhew how theſe queſtions may be reſolved by *Projection of the Sphear in plano.* And to do this,

1. I ſhall ſhew you how to *Project the Sphear in plano.*

2. How theſe Sphærical Triangles do naturally lye in the Sphear it ſelf: Thus Projected.

3. How the Sides and Angles of theſe ſphærical Triangles may be meaſured in the Projection by a Line of Chords only, whereby theſe queſtions are moſt ingeniouſly reſolved.

SECT. I.

How to Project the Sphear in plano upon the Plain of the Meridian.

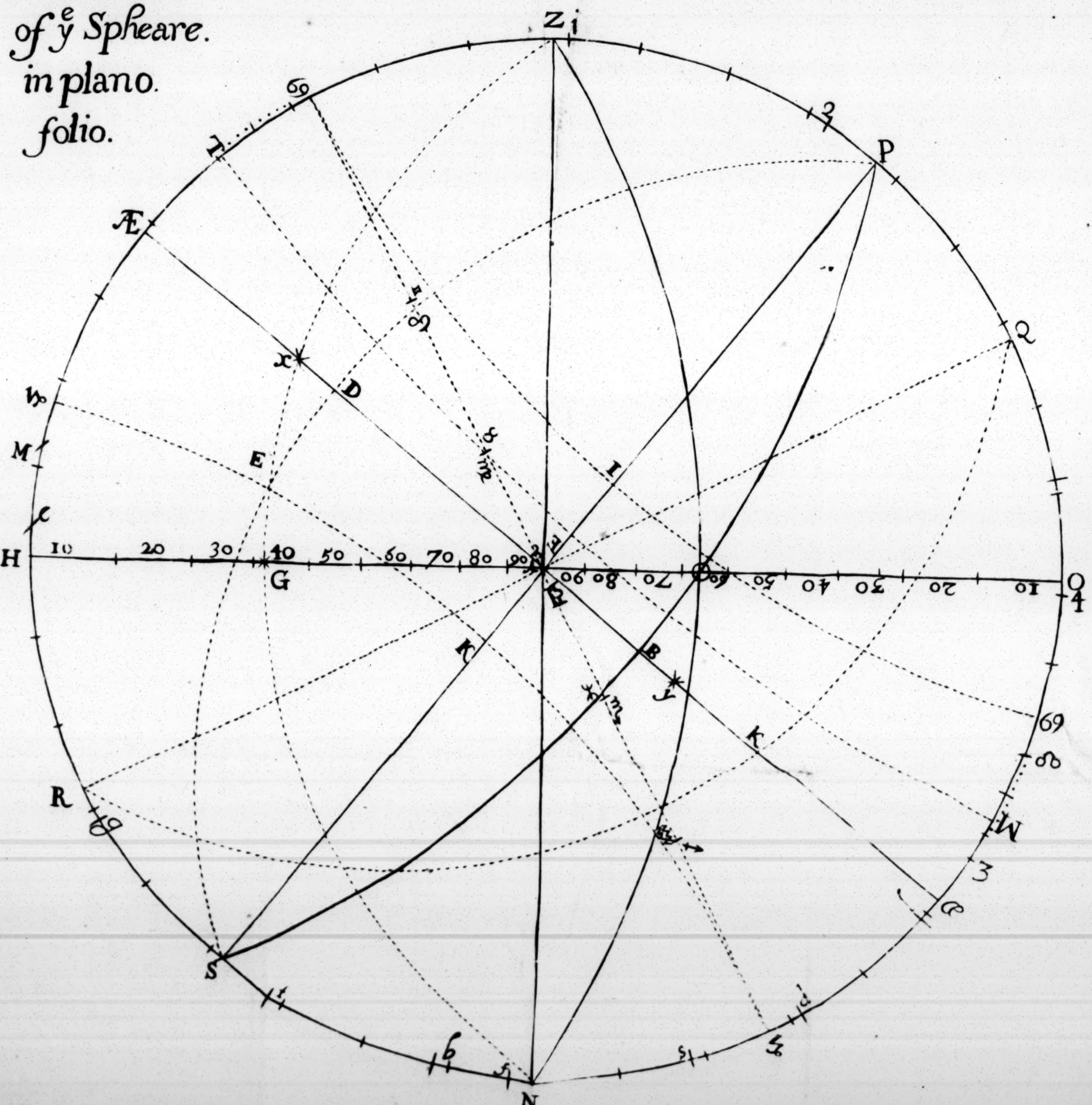
I might ſhew you ſeveral waies of Pro-

jection, as that which they call the Analemma, which is by drawing straight lines from one side of the Circle to the other, and thus you will find it projected by Mr. *Norwood* in his *Seamans Companion*, and in *Brown*, *pag.* 40. Also in Mr. *Gunter* very ingeniously: But that which is most generally used, is the Stereographick Projection, used by *Gemma Frisius*, which you will also find in *Brown*, *p.* 142, and in *Philips* Geometrical Seaman, *pag.* 75 which is the way used in Maps of the whole World: But that which includes all the rest in some respect, and which is most sutable to my present purpose, is that Projection of Mr. *Leyburns* in his Geometrical Exercises *pag.* 122. which I here follow, being to be done by a Line of Chords only, and upon which Projection by the Intersection or crossing of the several Circles thereof, are constituted divers sphærical Triangles: By resolving of which, most of these fore-mentioned Astronomical questions of the sphear may with speed and exactness, as also with much pleasure and delight be resolved.

First then, Take 60 Degrees or Radius from your Line of Chords, and with that distance upon the point A, as a Center describe the Circle Z H N O. representing the Meridian (within which all the rest are to be projected) and cross it with the two

Diame-

The Projection.
of y^e Spheare.
in plano.
folio.



between 338 and 339.

Diameters HAO, and ZAN, the former repreſenting the Horizon; the latter the Zenith and Nadir, or the prime Vertical.

Secondly, (Becauſe the Latitude of the place for which you draw your projection is 51 *d.* 30 *m.*) Take 51 *d.* 30 *m.* from your Line of Chords, and ſet them upon the Meridian from O to P, and from H to S, and draw the Line PAS, the Axis of the World P ſignifying the North Pole, and S the South Pole: Alſo with the ſame diſtance ſet off 51 *d.* 30 *m.* from Z to Æ, and from N to æ, and draw the line Æ Aæ for the Equinoctial.

Thirdly, Take 23 *d.* 30 *m.* the *Suns* greateſt Declination, and alſo the Diſtance of the 2 Tropicks from the Equinoctial, and ſet them upon the Meridian from Æ to ♋, and from Æ to ♑; and alſo from æ to ♋, and from æ to ♑: This done, draw a right Line ♋ A ♑ between the two Tropicks, touching the Tropick of *Cancer* above the Horizon, and the Tropick of *Capricorn* below the Horizon, and this is the Ecliptick: Then take 23 *d.* 30 *m.* out of your Line of Chords, and ſet them from P to Q, and from S to R, and draw the Line QR the Axis or Poles of the Ecliptick. Now to divide the Ecliptick into ſigns and degrees, take 60 *d.* from your Line of Chords, and ſet them from Q to 1, and from Q to 3: Alſo ſet the ſame diſtance

from ♋ to 2, and from ♑ to 4; this done, lay a Ruler to the Pole R, and the figure 1, it will cut the Ecliptick in the point ♊ and ♌: The Ruler laid to R and 2 will cut it in the points ♀ and ♍, and laid to R and 4 in ♏ and ♓; and laid to R and 3 in ♐ and ♒. And if you divide every of the spaces ♋, 1, 12, 2 Q, Q 4, 4 3, and 3 ♑ into 3 equal parts, each part will contain 10 degrees, and a Ruler laid to each of them, and the point R shall give you the points upon the Ecliptick answering to the 10 degrees of every sign, and in the same manner you may put on every degree. Thus you have drawn the Meridian Horizon, the prime Vertical, the Axis of the World, and the Æquinoctial, the Axis of the Ecliptick, and the Ecliptick, all which are streight lines: Now we come to draw those which are Circular, or Arches of Circles; and before you can draw these, the Centers of those Circles must be found out: Now I shall shew you two waies of finding out these Centers: One of these is Mr. *Phillips* in his *Geometrical Seaman*, on this manner, divide the Meridian into degrees in 4 Quadrants, and keeping one end of your Ruler fixed at the Point at Z: Lay the other end to the several degrees in the lower Semi circle HNO, so you shall divide the Diameter HO into 9 parts, which are half-

half-Tangents. In the same manner, if you keep one end of your Ruler fixed at the point H, and lay the other end to the several degrees of the Semi-Circle ZON. You may divide the Diameter ZN into 9 unequal parts, which are half-Tangents. Now having thus divided the Diameters, they must guide you in drawing of the Meridians, and Parallels which are all parts of perfect Circles: And you may find their Centers by these three points. First, For the Meridians, they all concur in both the Poles, and their third point is their correspondent degree in the opposite Diameter, and for the Parallels, two of their points are their degrees in the outward Circle, and their third point is their correspondent degree in the opposite Diameter: Further you must know that the Centers do all lye in the Diameter Lines, extended beyond the Circle, (if you seek for the Center of a Meridian exceeding 45 degrees) for the Diameter being divided into half-Tangents, if for every degree you account 2 beginning from the Center A, so you shall have the Centers of the Meridians, and if you set one foot of your Compasses in that Center, and open the other to the Point Z or N, it will pass through the Correspondent degree in the Diameter H. O. As for Example, If you would draw the Meridian

ZGN, which cuts the Diameter in 32 degrees, HG, count from the Center towards O, double ſo many degrees as is betwixt H and G, which is 64 degrees, and that is the Center to the Meridian, Z. G. N. *viz.* W. The like is to be done for finding the Centers to the Parallels, as you may eaſily apprehend: But if any of theſe Centers fall ſo far without the Circle, that your Compaſſes will not reach them, there is an Inſtrument call'd a Bow, which by help of one or more ſcrews may be extended to touch any three points which lye near in a ſtraight Line; or a Ruler bridled may do the ſame if it bend equally.

This way of Mr. *Phillips* finding the Centers I cannot but applaud as very ingenious, but do prefer that of Mr. *Leyburns*, which is thus: Upon a long piece of ſtiff paper deſcribe a Quadrant as ABC, and upon the point C erect an exact perpendicular. Let the Quadrant be divided into 90 degrees, and conſider how many degrees the Arch of the Circle (whoſe Center you ſeek) is diſtant from the Meridian or outward Circle, and a Ruler laid from the Center of the Quadrant by ſo many degrees in the Limb thereof to cut the perpendicular, where the Ruler cuts the perpendicular make a prick or mark, and the diſtance betwixt the ſaid mark and the Center of the Quadrant A ſhall

shall be the Semidiameter of the Circle sought: To make this plain by these following Examples, First, for the Tropicks, whose Distance from the Equinoctial is 23 *d.* 30 *m.* lay the Ruler to the Center of the Quadrant A, so as it cut 23 *d.* 30 *m.* from the point B in the Limb at *f*, and cuts the perpendicular at *g*, the Distance A *g*, or the Line A *f g* taken in your Compasses, and setting one foot upon the Polar Line extended till the other foot just reach the Equinoctial at Æ; draw an Arch of a Circle, and it shall be one of the Tropicks, as ♋ I ♋ the Tropick of *Cancer*, and ♑ *k* ♑ the Tropick of *Capricorn*: The like may be done by any other parallel of Declination, or Latitude, whether they fall nearer or further off the Equinoctial.

Another Example take in the Hour-Circle PBS, which is distant from the Meridian 62 *d.* 46 *m.* wherefore take 62 *d.* 46 *m.* from your Line of Chords, and set them from *a* to *b*, then laying a Ruler from P to *b*, it will cut the Equinoctial in B, through which point the hour-Circle of 62 *d.* 46 *m.* must pass. To find the Center belonging to this hour-Circle, lay your Ruler to the Center of the Quadrant at A, and let it pass by 62 *d.* 46 *m.* in the Limb: From the point C it will cut the perpendicular in M, therefore taking the Distance in A in your

Compasses, and setting one foot in the Equinoctial extended, and with the other just to reach P or S, strike an Arch of a Circle, and it will cut PBS the Hour-Circle of 62 *d.* 46 *m.* The like may be done in any other hour-Circle, or Azimuth-Circle, Parallels of Declination, Circles of Altitude, or any other whatsoever; only observe, that the Centers of all hour-Circles fall in the Equinoctial Line Æ A æ extended, the Centers of Azimuth-Circles in the Horizon HAO extended, where need is: The Centers of the Tropicks, and parallels in the Axis of the World PAS, and the Centers of the Circles of Altitude fall in the prime Vertical Circle ZAN.

Lastly, As every Circle in the Projection hath its proper Center, so hath it also its proper Pole. For the finding whereof, note that the Pole of a great Circle is 90 degrees, or a Quadrant distant from the Circle it self upon that Line which cutteth the Circle at Right Angles: Thus the Poles of all the Hour-Circles are upon the Equinoctial: The Poles of all the Azimuth Circles upon the Horizon, *&c.* Now if you would know the Pole of the Hour-Circle PDS, lay a Ruler upon P and D, and it will cut the Meridian-Circle in *e*: Then take 90 degrees from your Line of Chords, and set them from *e* to *f*, a Ruler laid from P to

to *f*, will cut the Equinoctial in Y, so is Y the Pole of the Hour-Circle PDS, the same Rule is to be observed in finding the Poles of the Azimuth-Circles: As if you would find the Pole of ZGN, lay a Ruler upon Z and G, it will cut the Meridian-Circle in *g*, then set 90 degrees of your Chord from *g* to *d*, so a Ruler laid from Z and *d* will cut the Horizon HAO in the point ☉, which is the Pole of the Azimuth-Circle ZGN; and by the same manner of work you may find the Poles of all the rest. As of PDS is Y, PCS, V; PAS, Æ or *æ*, PBS. ZGN, ☉; ZAN, H or O; ZFN, T; Z☉N, G; HAO, Z and N; ÆA*æ*, P and *S*; ♋ A ♑, Q and R; And thus much shall serve to shew you how to Project the Sphear *in plano*, and how to find the Centers and Poles of all the Circles.

SECT. 2.

In the next place I come to shew you how the forementioned Questions of the Sphear may be speedily and exactly resolved by the Resolution of several Spherical Triangles which naturally lye within the Sphear being thus projected: And of these in this following order: Having before shewn you how to measure the side or Angle of any Sphærical Triangle, *pag.* 344, 345.

PROP.

P R O P. 1.

The Distance of the Sun from the nearest Æquinoctial Point *(either* Aries *or* Libra*) given, To find his Declination.*

In the Projection this Proposition is to be resolved upon the Triangle A *k* ♒ ; wherein you have given, 1. The Side A ♒ 59 *d.* the distance of the Sun from the nearest Æquinoctial point *Libra.* 2. The Angle ♒ A *k*, 23 *d.* 30 *m.* which is the Angle that the Ecliptick makes with the Equinoctial, and is alwaies equal to the greatest Declination of the Sun, and you are to find the side *k* ♒, which to do by the Projection.

Lay a Ruler upon the Pole of the Circle R ♒ Q (which is at ♉) and the point *k*, it will cut the Meridian in the point *l* : Also lay a Ruler from ♉ to ♒, it will cut the Meridian in the point ♑, so the distance *l* ♑ being measured upon your Line of Chords, will contain 20 degrees, the Suns Declination, he being in 29 *d* of ♒.

P R O P. 2.

To find the Latitude, or the Side PO of the Triangle P ☉ O.

You need do no more when you are to measure any side of a Triangle, which lies in

in the Meridian it ſelf, than to take the ſame ſide in your Compaſſes and meaſure it upon your Line of Chords, as in this Propoſition. Take in your Compaſſes the Diſtance from O to P, and meaſure it upon your Line of Chords, and it will be found to contain 51 *d.* 30 *m.* the Latitude required.

P R O P. 3.

The Latitude of the place, and Declination of the Sun given, To find his Amplitude.

This Propoſition may be reſolved by two ſeveral Triangles, one is P ☉ O, the other A ☉ B. In the firſt you have given the ſide PO the Latitude, and P ☉ the Complement of the Suns declination, and you are to find ☉ O. To do this by the Triangle, A ☉ B, lay a Ruler upon Z, (which is one of the Poles of the Horizon) and to the point ☉ it will cut the Meridian Circle in *d*, and laid from Z to A, it will cut the Nadir point in N; ſo the diſtance N *d* will be found 33 *d.* 20 *m.* The Amplitude of the Suns Riſing or Setting from the true Eaſt or Weſt points of the Horizon. In the Triangle A ☉ B, you have given ☉ B the Suns Declination 20 *d.* The Angle ☉ AB, the Complement of the Latitude 38 *d.* 30 *m.* and you are to find the ſide ☉ A as above.

For

For finding the side ☉O in the Triangle P☉O, lay a Ruler to Z, the Pole of the Horizon, and the point ☉, and it will cut the Meridian Circle in D, so the distance *d*O measured upon your Line of Chords, will contain 56 *d.* 40 *m.* which is the Amplitude of the Suns Rising or Setting from the North point of the Horizon.

PROP. 4.

The Suns greatest Declination, with his Distance from the next Equinoctial point being given, To find his Right Ascension.

The Triangle A *k* ♒ resolves this proposition, wherein you have given, 1. The side A ♒ the distance of the Sun from *Libra.* 2. The Angle ♒ A *k* the Suns greatest Declination, and the right Angle at ♒, and you are to find A *k* the side of the Triangle. Lay a Ruler to P and *k*, it will cut the Meridian in *s*, and lay it again to P and A, it cuts the same in S, so the Distance *S s* will be found 56 *d.* 50 *m.* the Suns Right Ascension.

PROP. 5.

The Latitude and Declination given, To find the Ascensional Difference.

This is resolved in the Triangle A☉B, wherein you have given, 1. The side ☉B, the

the Suns Declination 20 *d.* 2. The Angle ☉AB, the Complement of the Latitude 38 *d.* 30 *m.* and the Right Angle at B, and you are to find the ſide AB. Lay a Ruler to P, the Pole of the World (and alſo of the Equinoctial) and the point B, it will cut the Meridian Circle in the point *b*, the diſtance *b. s.* meaſured, will be 27 *d.* 14 *m.* the Aſcenſional difference, which is ſo much as the Sun Riſeth or Setteth before or after ſix a Clock.

PROP. 6.

The Latitude, Declination, and Altitude of the Sun given, To find the Suns Azimuth either from the Eaſt, North, or South points of the Horizon.

That which is moſt difficult and intricate to perform by Numbers, is by Projection effected with the ſame eaſe as any of the reſt: As in this propoſition it is the Angle EZP, in the Triangle ZEP, which is here required: Lay a Ruler upon Z to the point G upon the Horizon, the Ruler thus laid, will cut the Meridian in G, ſo the diſtance *g*. O. meaſured upon the Line of Chords, will be 146 *d.* which is the Suns Azimuth from O the North part of the Meridian, and from *g* to H 34 *d.* which is the Azimuth from H the South-part of the Meridian, and

and from *g* to N 56 *d.* which is the Azimuth from A, the East and West point of the Horizon.

PROP. 7.

The Latitude, Declination, and Altitude of the Sun given, to find the hour of the day.

This Proposition is performed by resolving the Triangle Z P *a*, where you are to find the Angle Z P *a* : Wherefore lay a Ruler from the point P, to the point *a*, and it will cut the Meridian in *t*, so the Arch *t* Æ will be found 95 *d.* 52 *m.* which is the hour from the Meridian: The Arch *t a* being measured, will contain 84 *d.* 8 *m.* which is the hour from midnight : Also the Arch *t* S being measured, will contain 5 *d.* 52 *m.* the hour from six.

This Triangle Z P *a* is composed of Z P an Arch of the Meridian Z, *a* an Arch of an Azimuth Circle, and of P *a* the Arch of an hour-Circle.

PROP. 8.

The Declination, Altitude, and Azimuth of the Sun given, to find the hour of the day.

The Triangle ZEP in the projection resolves this Problem, wherein there is given

1. EP,

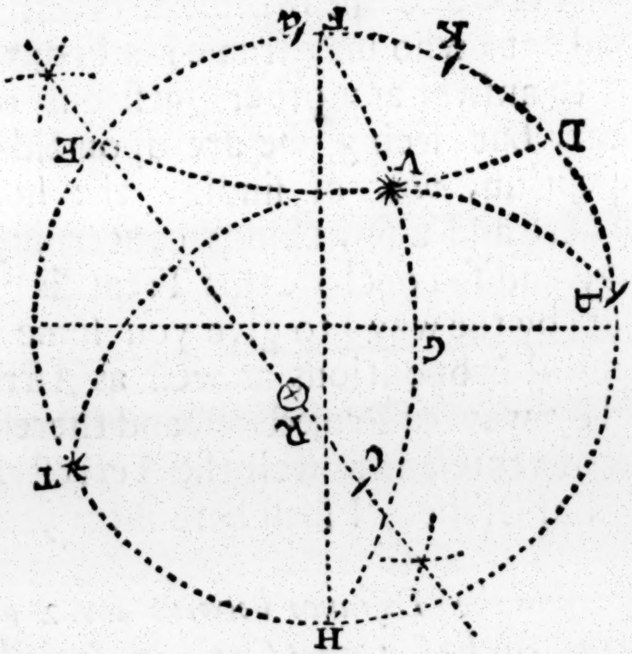
1. EP, the complement of the Suns Declination 70 *d.* South, 2. The ſide EZ the complement of the Suns Altitude 78 *d.* And 3. The Angle EZP the Suns Azimuth from the North. The Angle ZPE is to be ſound to anſwer this propoſition: Therefore lay a Ruler to the Pole P, and upon the point D in the Æquinoctial, and it will cut the Meridian in *e*. The diſtance *e* Æ being meaſured, will give you 35 *d.* 36 *m.* the hours from Noon which in time is 2 *h.* 22 *m.* Many more propoſitions might be added, which I leave to the ingenious practitioner to find out; who may frame his Projections ſo as to anſwer any other Queſtions of the Sphear, but ſeeing we are upon this laſt Propoſition, *viz.* of finding the hour, I ſhall here add a few things concerning Dialling, and ſo conclude this Treatiſe.

But (by the way) to give you ſome Geographical Propoſitions as well as Aſtronomical by way of Projection, and thereby to make it repreſent as well the Terreſtrial as Cœleſtial Globe. I ſhall here ſhew you,

How to find the Diſtance betwixt any 2 places by the Arch of a great Circle in Projection, howſoever diſtant from one another.

1. Draw a Circle, which may be ſuppoſed

sed to be the first Meridian, (which is reckoned from the Islands of Azores) cross this Circle through the Center with a Diameter, which is to be supposed the Equinoctial; If the two places differ only in Longitude, the Distance is to be measured upon the Equinoctial, or some other parallel: If they differ in Latitude only they are to be measured upon a Meridian; If they differ both in Longitude and Latitude, the Distance must be measured by drawing an Arch of a great Circle to pass through



both the places: And to measure any of those Distances, you must find the Pole to the great Circle which passes through both places:

places: How to do this, you have it elſewhere taught *pag.* 345: Then laying a Ruler to the ſaid Pole, and firſt to one of the places, and afterwards to the other, and marking at each time where the Ruler cuts the Meridian or outward Circle: The ſame diſtance taken in your Compaſſes, and meaſured upon your Line of Chords, gives in Degrees and Minutes the Diſtance betwixt the two places, which you know being multiplyed by 20, brings it into Leagues, or by 60 into Miles: I ſhall only give one Example for all: Suppoſe you would know the Diſtance betwixt the *Cape of Good Hope*, lying in the Latitude of 40 *d.* South and in Longitude 50 *d.* and *Malibrigo* lying in 26*d.* of North Latitude, and in 180 *d.* of Longitude you will find their diſtance to be 140 *d.* or 8400 miles by Projection thus, (H. T. E. F. K. D.)

Firſt, draw the Circle, and from the ſame Line of Chords by which you draw the Circle take 26 *d.* for the Latitude of *Malibrigo*, and ſet it from *a* upwards, and becauſe the Longitude is 180 *d.* there you muſt place it in the Meridian at T. which ſignifies *Malibrigo*: Now to place the *Cape of Good Hope* according to its Latitude and Longitude, do thus: Draw a Parallel of Latitude of 40 *d. viz.* DE, and a Meridian of 50 *d.* (H.G.F.) for the Longitude

gitude where these 2 intersect each other, is the place you design (the *Cape of good Hope*) there make a mark at V, and with the same Radius wherewith you drew the first Circle, find a Center which will cut T and V, which you may easily do by drawing a Line to pass exactly in the middle betwixt T and V, when that is done, you must find the Pole of the same Circle at R, and laying a Ruler from the Pole R to V, it will cut the Meridian in K. Lastly, Measure the Distance T K, and that gives you the degrees that one of the places is distant from the other.

Those that desire to be informed more at large concerning finding these Distances, I refer them to Mr. *Leyburn*, his Eighth Geometrical Exercise, *pag.* 131.

What is here said concerning finding these Distances, the ingenious may apply to Sailing by the Arch of a great Circle; but I shall leave that to the Readers own Study and Improvement.

To know how many Miles any place is distant from you, in whose Zenith the Sun, or any Star is at the same time.

Take the Altitude of the Sun, which suppose 53 *d* from your Zenith; and with Mr. *Norwood* account 69 Miles in a degree, multiply 53 by 69, makes 3657 and so many miles

Miles it is between the place where I now am, and the place in whoſe Zenith the Sun was at that time, and by this means you may know how far it is to any place over, which the Sun or Star is at any time.

CHAP. IX.

Of Dialling.

THE whole and compleat Art of *Dialling* is a ſubject ſufficient for a Treatiſe alone, and you will find ſeveral Authors have writ whole Books concerning the ſame, I ſhall confine my ſelf within the compaſs of theſe few leaves which are left me; but what ſhall be ſaid I hope will be plain and perſpicuous to any ordinary Capacity: Brevity, and withal perſpicuity being the Mark I aim at all along in this little Treatiſe.

There are many Denominations and Diſtinctions given to Dials, as *Horizontal*, *Vertical*, *Polar*, *Equinoctial*; *Erect*, *Direct*, *Inclining*, *Declining*, and *Reclining*, every way Eaſt, Weſt, North, and South, and many more, both Fixed and Pendulant: Plain and Globular within doors and without: It is not my intention to ſpeak to theſe particularly, but ſhall only ſhew you how to make an *Horizontal*, and an *Erect*, *Direct South Dyal*, which are moſt uſual, and moſt uſeful.

SECT.

SECT. I.

To draw an Horizontal Dyal by the Globe.

I have ſhewed you *pag.* 374, how to make the Globe it ſelf a Dyal, and by conſequence any round Ball : I ſhall now ſhew you how to make an *Horizantal Dyal* by the Globe.

Firſt then, Prepare your flat Dyal ground parallel to the Horizon, and thereupon draw a Meridian, which muſt repreſent the direct North and South, and croſs the ſame at right Angles, which muſt repreſent the Eaſt and Weſt. The firſt of which Lines is the 12 a Clock Line, the latter ſhews 6 a Clock both Morning and Evening then making the Interſection of theſe Lines the Center, deſcribe a Circle on the Dyal-ground to what diſtance you pleaſe, and divide the ſame into 360 *d.* (as all other Circles are) and ſubdivide the ſame into as many leſſer parts as you can : Being thus prepared, you are to find the diſtances of the Hour-Lines in this Circle for any Latitude of place, which to do, let the Globe be ſet to the Latitude deſigned, and then make choice of ſome of the great Circles in the Globe that paſſe through the Poles of the World, as for Example (the Æquinoctial colure) and apply the ſame to the Brazen Meridian, in which ſcituation it repreſents the Subſtile or 12 a Clock Line, or Meridia

idian Line (call it which you will) then urning about the Globe towards the West, ill 15 degrees of the Æquator have passed hrough the Brazen Meridian: You must nark the Degree of the Horizon which the ame Æquinoctial colure crosses, for that oint will shew the Distance of 1 and 11 Clock from the Meridian or 12 a Clock-Line: Then turning again the Globe forwards till other 15 degrees of the Equator do pass the Meridian, the same colure will point out the distance in the Horizon of the hours 10 and 2 from the hour of 12, and in the same manner you may find out the distances of all the rest of the hour-lines from 12 in the Horizon; but note that the beginning of these distances must be accounted from that part of the Horizon on which the Pole is elevated; Now these Distances of the hours being thus noted in the Horizon of the Globe, you must afterwards translate them into your plain allotted for your Dial-ground, reckoning in the circumference of it so many degrees to each hour as are answerable to those pointed out by the colure in the Horizon: Lastly, having thus done, the Gnomon or Style must be erected, where note that the edge of the Gnomon, which is to shew the hours by its shadow in all kind of Dyals, must be set parallel to the Axis of the World: That so it may make an

an Angle of Inclination with its plain ground equal to that which the Axis of the World makes with the Horizon, and that the Style is to ſtand directly North and South, and in the Meridian Line is a thing ſo commonly known, that it were needleſs to mention it.

Now if you would make an Erect South Dyal, know, That which is an Horizontal Dyal in one place, will be an Erect South-Dyal in another: As ſuppoſe you would make an Erect Dyal for the Latitude of 52: This is nothing elſe, but to make an Horizontal Dyal for the Latitude of 38 *d.* And if you make an Erect Dyal for the Latitude of 27 *d.* the ſame will be an Horizontal Dyal for the Latitude of 63 *d.* and the ſame proportion is to be obſerved in the reſt; and hence it appears that an Horizontal Dyal and a Vertical are the ſame at the Latitudes of 45 *d.* But thus much ſhall ſuffice to have ſpoken concerning drawing Hour-Lines by the Globe in an Horizontal or an Erect South Dyal.

SECT. 2.

The Analogy or Propoſitions to find them by are theſe.

PROP. 1.

For an Horizontal-Dyal.

As Radius or the ſine of 90 *d.* is to the ſine of the Latitude (ſuppoſe 51 *d.* 30 *m.*)

So

So is the Tangent of the hour, (*viz.* 15 *d.* which makes an hour in the Equinoctial.) To the Tangent of the Hour-Line from the Meridian, (*viz.* the distance of the hour of 1 from 12) which in this Example is 11 *d.* 51 *m*; And so is 30 *d.* the Tangent of two Equinoctial hours to the Tangent of the distance from 12 to 2, *&c.*

PROP. 2.

For an Erect-South Dyal.

As the Radius (90 *d.*) to the Co-sine of the Elevation (in this Example 38 *d.* 30 *m.*) So is the Tangent of any hour given, (suppose 15 *d.* for the hours 11 and 1, or 30 *d.* for 10 and 2.) to the Tangent of the hour-Line, (or to the true Distance of the hour-line) from the Meridian, (which in this Example is 9. 28, and 19. 46, *&c.*)

SECT. 3.

Here follows a Table taken out of *Seller*, shewing the distances of the Hour-Lines from the Meridian in several degrees of Latitude, both for an Horizontal, and also a South Erect Dial, as in the Latitude 54 the distance from 12 to 1 is 12 *d.* 41 *m.* from 1 to 2 is 25 *d.* 2 *m.* and so of the rest in an Horizontal plain: But in an South Erect Dyal the Distance from 12 to 1 is 8.56 from 1 to 2, is 18.45, and so in any other Latitude you will find the degrees answering to each hour, which is very rea-

dy. The Style or Cock of the Dyal must be fixed just over the Meridian Line, and must make an Angle with the Plain equal to the Height of the Pole, the hours before 6 in the morning, and after six in the Evening, may be supplyed by their opposite hours on the other side the Center.

A Table shewing the Distances of the Hour-Lines from the Meridian in these Degrees of Latitude.

Lat.	50	51.	52.	53.	54.	55.	56.	57.	58.	59.	
H.	D. M.	D. M.	D. M.	D. M	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	H
	2.52	2.54	2.56	2.59	3. 1	3.04	3.07	3. 8	3.10	3.12	
	5.44	5.50	5.55	6.00	6. 4	6.09	6.13	6.18	6.22	6.28	
	8.39	8.47	8.54	9. 1	9.10	9.15	9.21	9.27	9.34	9.43	
1.	11.36	11.45	11.55	12. 5	12.13	12.23	12.32	12.39	12.49	13.00	11
	14.33	14.46	14.57	15. 8	15.19	15.31	15.42	15.52	16.02	16.16	
	17.36	17.51	18. 3	18.17	18.29	18.45	18.57	19.08	19.21	19.38	
	20.39	20.56	21.11	21.27	21.40	21.57	22.12	22.24	22.38	22.58	
2.	23.51	24. 9	24.26	24.44	24.59	25.18	25.33	25.49	26.04	26.25	10
	27. 4	27.23	27.43	28. 3	28.19	28.39	28.56	29.12	29.29	29.53	
	30.23	30.44	31. 7	31.26	31.44	32.05	32.23	32.41	32.59	33.24	
	33.51	34.12	34.34	34.57	35.15	35.39	35.59	36.16	36.34	37.00	
3.	37.27	37.50	38.13	38.36	39. 3	39.18	39.38	40.00	40.17	40.44	9
	41.10	41.32	41.57	42.19	42.39	43.03	43.23	43.42	44.02	44 30	
	44.58	45.21	45.45	46.10	46.30	46.52	47.12	47.33	47.52	48.18	
	48.54	49.18	49.41	50.05	50.25	50.48	51.08	51.27	51.45	52.12	
4.	52.58	53.22	53.44	54. 7	54.26	54.47	55.05	55.25	55.43	56.09	8

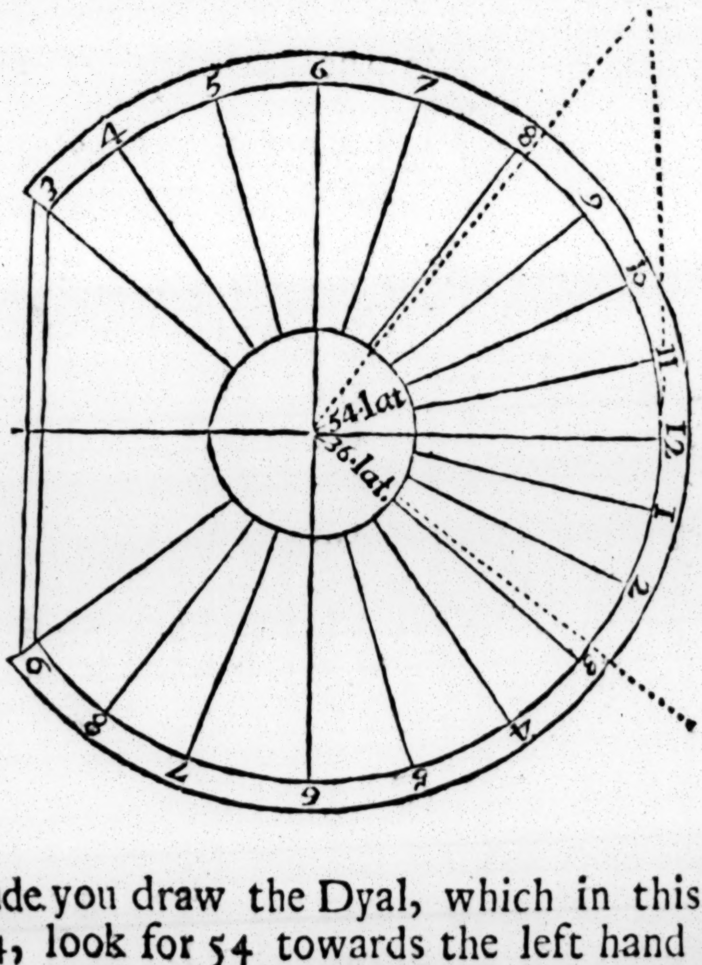
H.	50. D. M.	51. D. M.	52. D. M.	53. D. M.	54. D. M.	55. D. M.	56. D. M.	57. D. M.	58. D. M.	59. D. M.	H
	57.13	57.35	57.56	58.17	58.35	58.56	59.14	59.31	59.48	60.22	
	61.33	61.53	62.13	62.26	62.48	63.07	63.23	63.39	63.55	64.17	
	66. 5	66.23	66.40	66.57	67.11	67.28	67.42	67.56	68.10	68.29	
5	70.41	70.56	71.09	71.25	71.37	71.51	72.02	72.14	72.25	72.41	7
	75.26	75.37	75.49	76.00	76.09	76.20	76.29	76.35	76.47	76.59	
	80.14	80.23	80.30	80.38	80.44	80.52	80.58	81. 4	81.10	81.18	
	85. 2	85.06	85.10	85.14	85.18	85.22	85.26	85.31	85.34	85.39	
6	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.09	6
	40.	39.	38.	37.	36.	35.	34.	33.	32.	31.	

These Tables of Mr. *Sellars* are for an Horizontal Dyal, for Latitude 34 to 40, and from 50, to 59: The Complement whereof is for an Erect Direct South Dyal in such places of Latitude as is the Complement thereunto, as from 31 to 40, and from 50, to 56. This called by *Sellar* Tabular Dyalling easiest of any other.

South	56.	55.	54.	53.	52.	51.	50.	
Horiz.	34.	35.	36.	37.	38.	39.	40.	
	2.05	2.09	2.12	2.15	2.18	2.21	2.24	
	4.12	4.19	4.25	4.31	4.38	4.43	4.50	
	6.20	6.30	6.40	6.49	6.58	7.07	7.17	
1	8.31	8.44	8.57	9.09	9.22	9.34	9.47	11
	10.43	11 00	11.16	11.31	11.47	12.01	5.108	
	13. 2	13.21	13.41	14.19	14.35	14.55	14.00	
	15.22	15.46	16. 8	16.29	16.51	17.11	17.33	
2	17.52	18.18	18.44	19. 8	19.33	19.56	20.21	10
	20.27	20.56	21.24	21.52	22.19	22.45	23.13	
	23. 9	23 41	24.14	24.43	25.14	25.41	26.12	
	26.03	26 38	27.13	27.46	28.18	28.49	29.23	
3	29.11	29.49	30.27	31.01	31.36	32.08	32.44	9
	32.31	33.11	33.51	34.27	35.04	35.38	36.16	
	36.04	36.46	37.28	38.06	38.45	39.19	39.58	
	39.55	40.36	41.22	42.00	42.40	43.16	43,55	
4	44.03	44.48	45.29	46.10	46.48	47.24	48.04	8
	48.34	49.18	50.00	50.39	51.17	51.52	52.30	
	53.28	54.05	54.46	55.24	56.00	56.33	57.10	
	58.42	59.21	59.59	60.33	61.06	61.36	62.09	
5	64.20	64.53	65.27	65.56	66.26	66.52	67.20	7
	70.23	70.51	71.17	71-36	72.05	72 25	72,48	
	76.44	77. 8	77.22	77-38	77.38	78.09	78.25	
6	83.18	83.28	83.38	83-47	83.55	84.03	84.11	
	90.00	90.00	90.00	90,00	90.00	90.00	90.00	6

To draw an Horizontal-Dyal by this Table for 54 d. *Latitude.*

First, Take with your Compasses from a Line of Chords 60 *d.* with that distance describe a Circle, cross that Circle thorough the Center with a Diameter for a Meridian Line, and cross that at right Angles for the hour of 6, then consider for what Lati-



tude you draw the Dyal, which in this is 54, look for 54 towards the left hand of

this Table, and over againſt the Latitude, you will find 12 *d.* 14. for the diſtance of 11. and 1. from the Meridian, 25 *d.* 2 *m.* for the diſtance of the hours 10. and 2, *&c.* Theſe you are with your Compaſſes to take from your Line of Chords, and prick them upon the Dyal-Circle, and ſo draw Lines to each mark, which are the hour-lines. After you come at the hour of 6 you may draw the remaining hours, by laying a Ruler thorough the Center to their oppoſite hours on the other ſide the hour of 6. This needs no Example.

Note that for an Erect Direct South-Dyal, the Cock or Gnomon muſt be the Complement of the Latitude here 36.

Note alſo, That every Horizontal Dyal is a direct South Dyal in that place that is the Complement of the Latitude, as an Horizontal Dyal in 52 is an Erect South Dyal in 38.

Laſtly, To draw the Style, or Cock or Gnomon, take the degrees of your Latitude from the Line of Chords, and ſet it off ſidewaies from the hour of 12, and from the Center draw another Line to interſect the former through 54 *d.* which you ſee is near the hour-Line of 8. And thus you ſee how eaſie it is to draw the hour-lines by this Table for an Horizontal: The ſame may be done for an Erect South-Dyal, if you look

look for the Latitude on that side of the Table towards the right hand; there against 54 you find 8 *d*: 56 *m.* for 11. and 1. and 18.45. for 10. and 2. and so forwards; whereby you may understand to make an Erect South Dyal for any Latitude mentioned in the Table: If you please to do it by Arithmetical operation, you have also the Analogy, or Proportions to work them by; or if you please to do it by the Globe, you may find the hour-lines by that also as is shewed in the former pages, 407. And this is all at present I intend to say concerning finding the hour of the Day by the Sun, or making Sun-*D*yals: I shall only shew you how to find the hour of the Night by the Moon and Stars, and so bid good Night, and conclude this Mathematical Manual.

SECT. 4.

First then, By the Moon.

You must understand, that by knowing the Prime you may find the Epact, and by knowing the Epact you may find the Age of the Moon; by knowing the Age of the Moon, you may find the time of her coming to the South; and by knowing the time of her coming to the South, you may find the hour of the Night by the shadow of the Moon upon a Sun-*D*yal. Therefore of these in order.

P R O P. 1.

To find the Prime or Golden Number, Divide the year of our Lord by 19, and to that which remaineth after the diviſion add 1, the Product is the Prime Number for all that year, beginning the 1*ſt. January*: As

1673⌊88 $\frac{1}{19}$, ſo $\frac{1}{2}$ 2 is the Prime. (with 1 above the $\frac{1}{2}$)

$\cancel{1}\,9\,\cancel{9}$

$\cancel{1}$

P R O P. 2.

By the Prime, To find the Epact.

Multiply the Prime of the year by 11, and Divide the Product by 30, and the Remainder or Fraction is the Epact. If it be juſt 30, then is the Prime and the Epact both one; if ſhort of 30, that Number whatever it is, is the Epact. Note that the Prime changes 1. *January*, and the Epact 1. *March*.

P R O P. 3.

By the Epact, To know the Age of the Moon.

Add to the day of the Month the Epact, and ſo many dayes more as are Moneths from *March* to the Moneth you are in, including both Months; and if they come not to 30, ſo much the Moons Age: If they paſs 30, Take away 30, and the Remainder is the Moons Age. This is when the Month hath 31 dayes, but if it have only 30 dayes, deduct 29 *&c.* from the exceſs above 29, by the ſame reaſon.

PROP.

PROP. 4.

By the Age of the Moon, to find the time of her coming to the South.

Multiply the Moons Age by 12, and divide the Product by 15, the Quotient will ſhew the hour of the Moons coming to the South; and if any thing remain, multiply that by 4, and it will ſhew the Minutes to be added to the hours of the Quotient, and ſo you ſhall have the time of the Moons coming to the South: Or by the Table following you may readily find the time of the Moons coming to the South.

Dayes

The Dayes of the Moons Age.

Days	H.M.
1	0.49
2	1.38
3	2.26
4	3.15
5	4. 3
6	4.53
7	5.41
8	6.30
9	7.19
10	8. 8
11	8.56
12	9.45
13	10.34
14	11.23
15	12.11
16	
	After-noon.
17	1.10
18	1.49
19	2.38
20	3.26
21	4.15
22	5. 3
23	5.53
24	6.41
25	7.30
26	8.19
27	9. 8
28	9.59
29	10.45
30	11.34
	Morning.

The Hours of the Moons Age.

Hours	H.M.
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20
11	22
12	24
13	26
14	28
15	30
16	32
17	34
18	36
19	38
20	40
21	43
22	45
23	47
24	49

The uſe of the Table according to the Age of the Moon, Take the Hours and Minutes oppoſite thereunto in the Table; adding for every hour 2 Minutes more for her mean motion; and the Total ſhall ſhew the hour of the Moons coming to the South.

For Example.

Suppoſe the Moons Age be any day at Noon 10 days and 8 hours old. Look in the Table, and againſt 10 dayes you find ———————— 8 *h.* 8 *m.*
for the 8 odd hours — 16 *m.*
To which 8 Hours 24 *m.* allowing 2 Minutes for each Hour for her mean motion is —-17 *m.*
All which added together ſhews ſhe comes to the South at ———— 8 *h.* 41 *m.*

To

To know at what hour the Moon comes to the South, by the following Table.

Dayes after the Change or Full the Moon is South at.

0	1	2	3	4	5.	6.	7.
12	12.48	1.36	2.24	3.12	4.0	4.48	5.36

Dayes after either of the Quarters the Moon is South at.

6	6.48	7.36	8.24	9.12	10 0	10 48	11.36

To know the Hour of the Night by the Moons shadow upon a Sun-Dyal.

☽	H.	M.	☾
1	0	0	29
2	10	24	28
3	9	36	27
4	8	48	26
5	8	0	25
6	7	12	24
7	6	24	23
8	5	36	22
9	4	48	21
10	4	0	20
11	3	12	19
12	2	24	18
13	1	36	17
14	0	48	16
15	0	0	15

First know the Age of the *Moon*, and if it increase, from the shadow substract these hours and Minutes: If it decrease, add the same to the shadow, and it will give you the true hour.

PROP.

PROP. 5.

By knowing the time of the Moons coming to the South, To know the time of the Night by the ſhadow of the Moon upon a Sun-Dyal.

Obſerve by a Sun-Dyal as if you would ſee what a Clock it were by the Moon, and obſerve how much the ſhadow of the Moon doth either want, or is paſt the hour of 12 upon the Dyal: For ſo much it doth want of, or is paſt the time of the Moons coming to the South. *As for Example*,

Suppoſe as before the Moon did come to the South at 8 hours, 41 minutes after noon or at night, and the ſhadow of the Moon upon the Dyal were at 10. This wants 2 hours of 12, and therefore it wants 2*h*. of 8*h*. 41*m*. ſo that it is ſix of the Clock, and 41 *m*. But if the ſhadow had been at 2 upon the Dyal, then you muſt have added 2 *h*. to the Moons coming to the South, and ſo it had been 10 *h*. 41 *m*. at night: The like is to be done by any other hour whatſoever.

SECT. 5.

To know the hour of the Night by the Stars.

1. To do this by the Globe, Rectifie the Globe to the Latitude of the place, and bring the *Suns* place to the Brazen Meridian, and the Index of the hour Circle to 12: Then bring any ſtar to the ſame Altitude on the Globe that it hath in Heaven, and the

the Index of the hour-Circle will point to the hour of the Night.

2. *By Mr.* Gunters *Nocturnal.*

This Nocturnal is a very easie and ready way to find the hour of the Night by the stars wherein the Center upon which it moves is supposed the North-Pole, having several constellations near the same Pole described upon the Nocturnal, with a Circle of Moneths fixed lying without the Rundle, as also an Hour-Circle of twice 12 *h.* one of the 12 signifying 12 at Noon, the other 12 at Midnight; and the line supposed to be drawn betwixt 12 and 12, thorough the Center is the Meridian: Now for the use thereof, first look up to Heaven to the North Pole, and observe what stars are upon or near the Meridian in Heaven: Then place the same star to the like scituation upon the Rundle, and against the day of the Moneth you will find the hour of the Night:

3. *In the third place, To find the hour of the Night by the Stars coming to the Meridian; their Rising, and Setting, and Right Ascension.*

How to find the Right Ascension of any star by the Globe is taught, *pag.* 363, and is no more but instead of bringing the Suns place to the Meridian, you must bring the star, (whose Right Ascension you seek) to the

the Meridian, and the number of degrees contained betwixt the vernal colure and that degree of the Æquinoctial which comes to the Meridian with the ſtar, is that ſtars right Aſcenſion: Alſo by the Globe to find the time of any ſtars Riſing or Setting, or coming to the Meridian, is taught *pag.* 364. 364, How to find the Right Aſcenſion of any ſtar by Theakers Planiſphear is ſhown *pag.* 374: As alſo their riſing and ſetting, and their coming to the Meridian, is ſhown *pag,* 375.

To find the Suns Right Aſcenſion is the ſame both by the Globe and Planiſphear, by bringing the Suns place to the Meridian in ſtead of bringing the Star to the brazen Meridian on the Globe, or the paper Meridian of the Planiſphear, as is explained in the forementioned places, *pag.* 362, 374. you have alſo the Analogy or Proportion to work it by Arithmetically, *pag.* 350. So need not here repeat any of them.

But ſhall ſhew you how by the Right Aſcenſion of the Sun and ſtars to find Arithmetically the time of the ſtars coming to the South, and thereby the ſtars hour, and by all theſe the hour of the night.

1. *To find at what hour any Star will be upon the Meridian.*

Add the Complement of the Suns Right Aſcenſion for the day propoſed to the Right Aſcenſion of the Star, the ſum of them together will be the time of that ſtars being upon the Meridian.

For

For Example.

	h.	m.
The Bulls Eye 1*st*. *January* Right Aſcenſion.——	4	17
Compl. of the Suns Right Aſcenſion is————	4	25
The Bulls Eye upon the Meridian.—-————	8	42

That is, 42 Minutes paſt 8 at Night.

It is noted by *Norwood* that any ſtar in 14 dayes comes to the Meridian juſt an hour later, and in a Moneth two hours.

2. *To find the Riſing and Setting of the Stars.*

The Semidiurnal Arch of any ſtar, which is half the time the ſtar appears above the Horizon, ſubſtracted from the time of the ſtars being upon the Meridian, giveth the time of the ſtars Riſing, and the ſame being added thereunto, giveth the time of the ſtars ſetting; and knowing the time of the ſtars riſing or ſetting, and coming to the Meridian; and ſeeing any of the ſaid ſtars riſe, ſet, or come to the Meridian in Heaven, hereby you know the Exact hour of the Night.

3. *But to find the hour of the Night at any other time, you muſt know the ſtars hour.*

Now the ſtars hour is the horary diſtance, or the number of hours that the ſaid ſtar is diſtant from the Meridian, or ſo many hours as the ſtar is ſhort of, or paſt the Meridian.

Now

Now to find the Hour of the Night at any time, by any of the Stars.

Add the Stars Hour, the Stars Right Aſcenſion, and the Complement of the Suns Right Aſcenſion; all three together, the Sum is the hour of the Night. As for Example.

The Bulls Eye 11*th* of *December.*

The Stars Hour is	8	56
The Stars Right Aſcenſion is	4	17
The Complement of the Suns Right Aſcenſion.	6	9
Somme	19	22

Now becauſe this exceeds 12, you muſt deduct 12 from it; ſo that the true hour of the Night is 7 *h.* 22 Minutes: And note further, That when this Sum is ſhort of 12, it ſhews the hour at Night; when it exceeds 12, it ſhews the hours next Morning: So that in this Example it is 7 *h.* 22 *m.* in the Morning; and herewith I ſhall conclude.

Having finiſhed what I intend to ſay concerning *Aſtronomy*, and the Reſolution of theſe Aſtronomical Queſtions of the Sphear, being the third and laſt Mathematical Liberal Science according to my Diviſion of them in the beginning: I ſhall therefore here ſet up a *Non Ultra* at this time. Having (I hope) in ſome meaſure ſhewed you, How to Meaſure the *Earth*, the *Sea*, and the *Heavens*, principally and eſpecially by *Trigonometry*. Therefore in alluſion to a *Triangle*, the moſt excellent of Geometrical Figures, and the other *Ternaries* mentioned here, and elſewhere, *pag.* 2, 3. I ſay in alluſion to theſe. To the Tri-vn Deity. The Trinity in Unity, and Unity in Trinity, who created All theſe Things of Nothing in Number, Weight, and Meaſure, by his Infinite Wiſdom and Power, and doth Uphold and Govern All by his Providence, and deſign'd all for his Glory, to him be all the Glory and Honour, and Adoration *in Secula Seculorum.* AMEN.

FINIS.

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